TEXT FLY WITHIN
THE BOOK ONLY

UNIVERSAL
LIBRARY

COMETS
AND METEOR STREAMS

THE INTERNATIONAL ASTROPHYSICS SERIES

VOLUME TWO

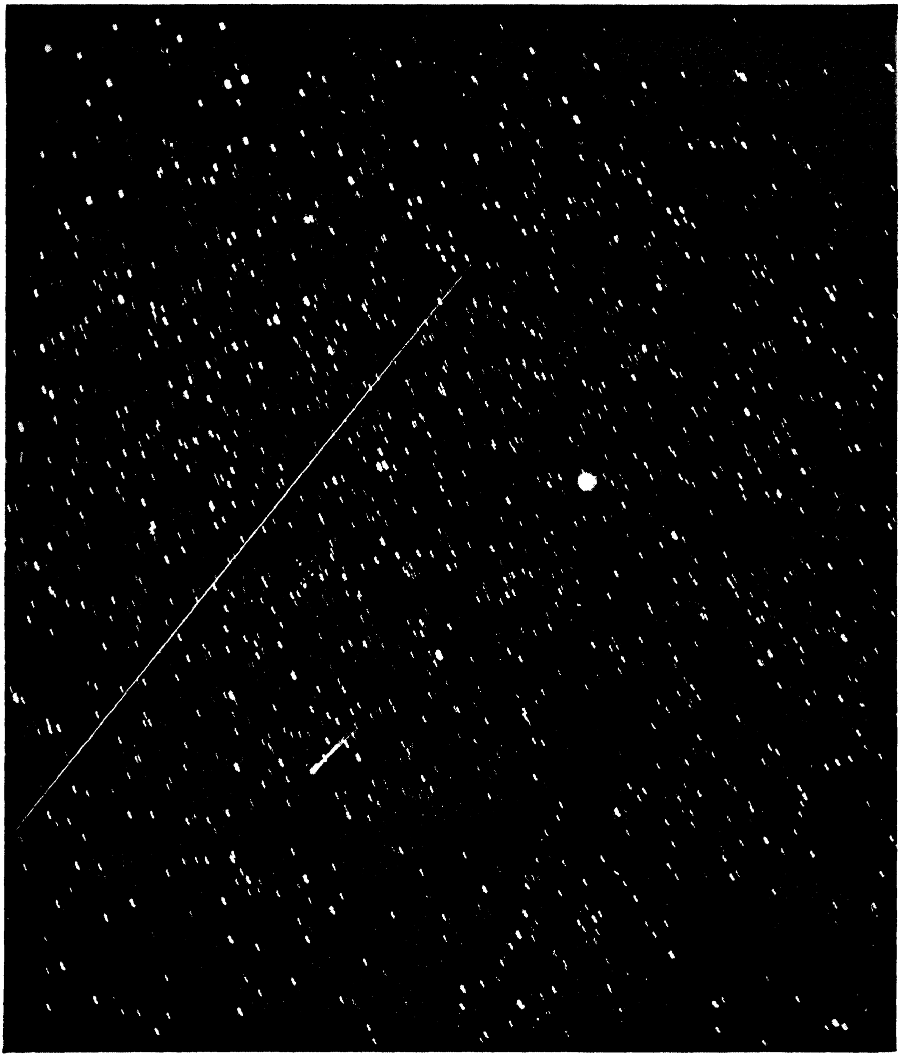
THE INTERNATIONAL ASTROPHYSICS SERIES

Already published

The Aurorae	*L. Harang*
Comets and Meteor Streams	*J. G. Porter*

In preparation

Astronomical Photometry	*D. S. Evans*
The Earth and the Planets	*W. H. Ramsey*
Stellar Constitution	*D. H. Menzel and H. K. Sen*
Close Binary Systems	*Zdenek Kopal*
General Relativity and Cosmology	*G. C. McVittie*
Gaseous Nebulae	*L. H. Aller*



COMET BROOKS, 1893 IV
and a meteor trail

(*Barnard, Lick*)

Frontispiece

THE INTERNATIONAL ASTROPHYSICS SERIES

VOLUME TWO

Comets and Meteor Streams

J. G. PORTER
Ph.D., B.Sc.
Principal Scientific Officer
H.M. Nautical Almanac Office

LONDON
CHAPMAN & HALL LTD.
37 Essex Street, W.C.2
1952

First Published 1952

Catalogue No. 440/4

PRINTED IN GREAT BRITAIN
BY W. & J. MACKAY & CO., LTD., CHATHAM

Editors' Note

The aim of the International Astrophysics Series is to provide a collection of authoritative volumes dealing with the main branches of Astrophysics and Radio Astronomy. The need for such a series of books has arisen because of the great developments which have taken place in these fields of work during recent years.

The books will be suitable for both specialists and students. Some of the titles may have a wider and more popular appeal, but this will be secondary to their main purpose, which is to assist in the teaching of Astrophysics and Radio-Astronomy and in the advancement of these subjects themselves.

Author's Preface

This book had its origins in the Physical Society Conference held in Manchester in 1947 to discuss the new radar methods of investigating meteors, and their bearing on related problems ; and it is, in effect, an expansion of a brief lecture that I gave on that occasion. The results of the new technique, and the revival of interest in the structure and origin of comets, have given us a new insight into the nature of the solar system, and presented us with further problems. It is my firm belief that one of our outstanding problems is that of the dynamics of the solar system. Indeed, it is now realised that the motions of comets and planets cannot be represented with that exactness which gravitational theory was believed to be capable of giving. There is little doubt that the study of celestial mechanics, which reached its highest development in the hands of Laplace (and which is now almost completely neglected in this country), is about to enter a new phase. It is certainly impossible to consider any study of comets and meteors which does not involve the paths in which they travel, and for this reason I have laid the main emphasis in this book on the orbits of these bodies and the changes to which they are subject.

Although a general account has been given of the main topics of interest in connection with these bodies, it has been found impossible to cover the descriptive parts of the subject in any detail. Thus there is no reference to the study of meteorites, so actively pursued to-day in America, nor is there more than a brief mention of the methods of the new radar technique. On the other hand, the statistics of comet orbits has been treated in detail, and accounts are given of the methods used in computing perturbations and in deriving the orbit of a meteor stream. This last calculation, which is comparatively simple, affords a convenient approach to the more difficult task of computing the orbit of a comet from three observations. The majority of the mathematical formulae which I have used are derived *ab initio*, and new methods have been developed in many cases. Much of the material of Chapters 4 and 6 is original, and is published here for the first time. At the end of each chapter I have given a list of books, lectures and reviews, as well as references to original work ; and these, although by no means exhaustive, are mainly in English, and are all available in this country without difficulty. The reader will find in this bibliography sufficient references to further reading in a literature which is already

immense. Acknowledgement is made to the British Astronomical Association for permission to reproduce the photographs of the " Eclipse " comet of 1948 and of Peltier's comet from the *Journal* of the Association. These, and the other photographs, have been chosen to illustrate some of the many variations which occur in the appearance of comets.

This book owes much to my wife, who has not only drawn all the diagrams, but has continually given me her interest and encouragement.

J. G. P.

Contents

List of Plates

CHAPTER I

Comets: General

The early history of cometary discovery is largely a record of their unexpected appearances. Unpredictable, they seemed to avoid the Zodiac, where the planets were always to be found ; and beyond the obvious fact that a comet had a tail, often described in the most imaginative terms by the observer, no truly scientific fact emerged for many centuries. It was natural that the occurrence of national or domestic disasters should be dated by reference to the appearance of such remarkable apparitions, and so in the course of time, superstition rivalled sound observation. " Old men and comets," wrote Dean Swift, " have been reverenced for the same reason ; their long beards, and pretences to foretell future events."

That the tail of a comet always points away from the Sun must have been known to many of the ancient observers, but it first seems to have been proved conclusively by Peter Apian in 1531. Tycho Brahe, observing the comet of 1577, was the first to show that a comet was more distant than the Moon. Tycho assumed that the orbit of a comet was circular, but Kepler, in the early years of the following century, does not seem to have associated the comets with the Solar System. He made no attempt to apply his three laws of planetary motion to the comets, but treated their motion as rectilinear. Modern ideas of the nature of cometary orbits may be said to have had their origin in the discussions of that remarkable group of early Fellows of the Royal Society, Robert Hooke, Christopher Wren and Edmond Halley. Halley's friendship with Newton had led to the publication of the *Principia* in 1686 ; later Newton had shown how the parabolic orbit of a comet might be computed, and Halley, with characteristic energy, collected the data of twenty-four comets, and computed their orbits. Struck by the similarity of the orbits of the comets of 1531, 1607 and 1682, he made every possible comparison between them, and deduced that they were really one and the same body, travelling, like the planets, in an elliptical orbit about the Sun. Making some rough estimates of the perturbing effect of Jupiter on the comet's motion, he concluded that the comet would return at the end of 1758. " Wherefore if it should return according to our prediction about the year 1758, candid posterity will not refuse to acknowledge that this was first discovered by an Englishman." The comet was recovered on Christmas Day, 1758,

sixteen years after Halley's death, and made its perihelion passage on 1759 March 12. Posterity has indeed recognised the greatness of the achievement—the first prediction of its kind—by calling this Halley's comet.

Nomenclature

Comets are usually named after the discoverer, but the rule is occasionally broken, if it is deemed necessary to honour some particular astronomer who, like Halley, has worked on the problems attached to a particular comet. An outstanding example of this is the case of Encke's comet, the

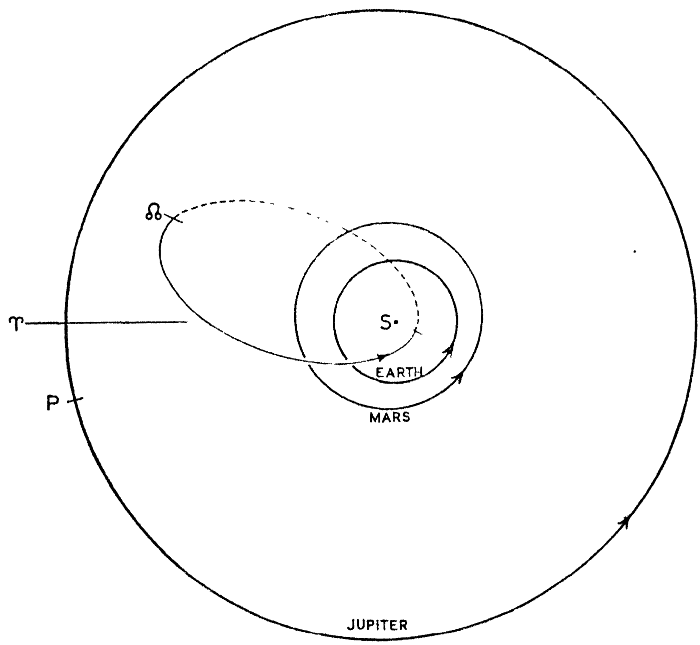


FIG. 1.—The orbit of Encke's comet.

history of which illustrates clearly the rapid advances which were being made in celestial mechanics in the early years of last century. The discovery of the minor planets had called for more accurate methods of computing the paths and perturbations of these bodies, if they were to be recovered after conjunction with the Sun. A method for the computation of elliptical orbits was evolved by Gauss in 1802, and the success which attended these new developments in mathematics was soon to be followed by their application to the paths of comets. In 1818 Pons at Marseilles discovered a faint comet, and Encke, a pupil of Gauss, undertook to calculate an orbit for the new object. It proved to revolve about the Sun in 3·3 years, a period which still remains the shortest of any known comet.

Encke, however, proceeded to show that the comet was identical with Méchain's comet of 1786, with Caroline Herschel's of 1795, and with Pons' discovery of 1805. All of these orbits had previously been computed as parabolas, but Encke proved them to be one and the same object, and after weeks of laborious work, was able to predict its next return on 1822 May 24. The comet was recovered at Paramatta close to the predicted place. " The importance of this event," says Agnes Clerke, " can be better understood when it is realised that this was only the second instance of the recognised return of a comet . . . and that it moreover established the existence of a new class of celestial objects, somewhat loosely distinguished as comets of short period."

In modern times we have a gratifying example of this re-naming of a comet in the object which for many years was known as Comet Pons-Coggia-Winnecke-Forbes. This unusually complex name (only three names are applied to a comet in cases of multiple simultaneous discovery) owes its origin to Crommelin's work. Comet Forbes of 1928 was shown by Crommelin to have an elliptical orbit, and to be similar to the orbits of a comet discovered in 1818 by the indefatigable Pons, and the comet of 1873, discovered independently by Coggia in Marseilles and Winnecke in Strasbourg. By a long series of calculations, which culminated in 1936, Crommelin was able to prove that all three appearances were actually those of the same object, a comet with a period of 28 years, but which had been seen on these three occasions only. Crommelin's work was recognised on the continent by the frequent references which appear in the literature to " Crommelin's Comet," but it was not until the 1948 meeting of the International Astronomical Union that this title received official recognition. The new name of this comet commemorates in a fitting manner the work of one of the greatest of all cometary astronomers.

In addition to having a name, a comet is also given a number. When first announced, the comet is numbered by the year of discovery followed by a small letter which indicates the order of discovery in that year. Later, when a satisfactory orbit has been computed, the order of perihelion passage is used. The year of perihelion passage is now adopted, and is followed by a Roman numeral to indicate the order of perihelion passage in that year. This designation is subject to the approval of the appropriate Commission of the I.A.U. As an example of this system, we may instance Comet Whipple. The second discovery in 1940, it thus became 1940b, but in order of perihelion passage, it was actually the third comet in 1941, and thus became 1941 III. As a periodic comet, its nature is designated by the prefix P/, so that it is fully described as 1941 III P/Whipple. Similarly Comet Kulin 1940a became 1939 VIII P/Kulin. The difficulty of dealing with comets which bear the name of the same discoverer is

overcome, in the case of periodic comets, by adding an Arabic numeral after the name, no such addition being required in the case of comets which are not likely to return. As an illustration of this ruling, the names given to the comets discovered by du Toit at Bloemfontein may be quoted from a resolution of the I.A.U. at the 1948 meeting. The comets are now known as 1941 VII P/du Toit-Neujmin-Delporte ; 1944 III P/du Toit (1) ; 1945*c* P/du Toit (2) ; 1945*d* du Toit ; 1945*g* du Toit. It may be noted that the last three comets mentioned will probably become known as 1945 II, 1945 III and 1945 VI.

Cometary discoveries

The number of comets discovered during any one year has naturally increased with the development of the telescope and the use of photography. It is usually considered that some five or six comets are discovered, on the average, each year, but the number varies considerably. Thus in 1931 there was only one cometary discovery, while 1932, 1947 and 1948 each brought thirteen comets in view. Some of these are seen for a short time only, while others may be visible for so long a time that the discoveries of one year overlap those of the next. In 1948, for instance, no less than twenty-four comets were under observation, but only one of these was at all conspicuous. The number of naked eye comets, in fact, may be estimated at only twenty or thirty in a century. In the remote past, only these more conspicuous comets would have been recorded, and more particularly those which could be associated with stirring events in world-history. In the late eighteenth and early nineteenth century, it became common for amateurs with small telescopes to search the sky for comets. The instrument employed was usually a small telescope with a lower-power eyepiece giving a wide field, and such a glass in the hands of an enthusiast like Pons or Messier would soon yield results. The same method is usefully employed to-day, although many comets are discovered by photography in the course of routine examination of the sky at the larger observatories. The first successful photograph of a comet was secured in 1881 with the new dry plates, while Barnard's comet 1892 V was the first to be discovered by this means. The rapid increase in cometary discovery brought about by the use of larger telescopes and of photography, is illustrated in Fig. 2, which gives the number of comets discovered in ten-yearly periods. The influence of Pons and Messier in the period 1760–1830 is apparent, and the great strides made during the active years of the mid-nineteenth century will be seen in this diagram. It will be noticed that the upward trend in numbers has been balanced by an improvement in accuracy, which is shown by the increasing proportion of comets for which elliptical orbits have been computed.

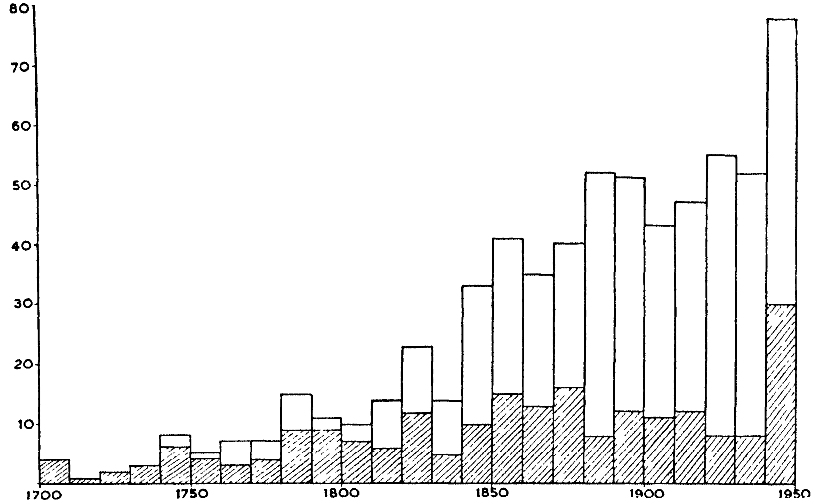


FIG. 2.—Growth of cometary discoveries, 1700–1950. (The shaded areas represent parabolic orbits.)

Despite the growth in cometary discovery, the situation is not as satisfactory as it might be. Although to-day there are one or two large telescopes in the southern hemisphere which are occasionally used for cometary observation, the number is far too small for a complete watch to be kept on the whole sky. Furthermore, few of the major observatories care to interrupt an already overcrowded research programme to undertake observations of a faint comet. Fortunately some of the large instruments on the American continent are used regularly for this work, but the need for more observations is acute. The recent use of the large Schmidt cameras opens up fresh possibilities, and there is little doubt that the new advances in optical instruments will have important results in cometary work. The average comet is seen for only a short time, and during that time is frequently very near the Sun, so that observation is difficult. It is all the more necessary, therefore, that every available opportunity should be taken for the study of these objects. The main factors in the slow progress that has been made are the limited opportunities for study of comets, and the comparatively few observers who care to undertake this work.

Appearance

In appearance, the average comet is devoid of structure, a nebulous object with little or no central condensation, and too faint for spectroscopic examination. Its measured position is therefore liable to error, for the centre of the diffuse patch of light does not always coincide with the centre of mass. The brighter comets have naturally provided most of our knowledge of these objects, and in the telescope, the head of such a

bright comet often shows a considerable amount of detail. No two comets are alike in appearance, but the head of a bright comet may be said to consist of a *nucleus*, which, when visible, has an almost stellar appearance, and is surrounded by a nebulous *coma* which merges imperceptibly into the *tail*. The coma may be seen to take the form of concentric shells or envelopes of approximately parabolic outline, while jets or fans of luminous matter are sometimes seen, directed from the nucleus towards the Sun, and then sweeping back to form the shells of the coma. Double or multiple nuclei are occasionally seen, and these may be of variable brightness.

As the comet continues to approach the Sun, the tail lengthens and the brightness increases, while the coma diminishes in size. Comets frequently attain their maximum brightness just after perihelion passage, and extensions of the coma *towards* the Sun may be seen about this time. In the brightest comets, the tail, if not seriously foreshortened, may assume magnificent proportions, and its full structure can best be studied from photographs. Multiple tails are not uncommon, while condensations of luminous matter within the tail may be seen in rapid motion away from the Sun. The reality of this motion must be studied from a knowledge of the distance of the comet from the Earth, for the apparent motion is no guide to the true state of affairs. Thus the rapid undulations in the tail—" like a torch agitated by the wind "—which are often seen, must be atmospheric effects ; for any movement along a comet's tail, however rapid, would appear to take several minutes for completion as seen from the earth, owing to the time taken for light to reach the observer from the different parts of the comet's tail.

Mass and Size

Both the head and the tail of a comet are, as a rule, completely transparent, and although some observations have suggested a diminution of the light of a star as the comet passes in front of it, no such effect is observed in the majority of cases. The comet of 1882 and Halley's comet of 1910 both passed between the Sun and the Earth, yet nothing was seen of either of them during the transit across the Sun's disk. The Earth has on occasions passed through the tail of a comet without any untoward effects beyond a suggested auroral glow in the sky on the occasion of the passage through the tail of Halley's comet in 1910. Nothing could be more tenuous than the whole structure of a comet, yet the nucleus must possess mass, since it obeys the law of gravitation. The actual mass of this nucleus must be very small. Lexell's comet of 1770 passed within the orbits of, and very close to, the satellites of Jupiter without disturbing them, and the same comet came within 1·5 million miles of the Earth without causing a

change of any kind, as, for instance, in the length of the year. These and similar facts enable us to set an upper limit to the mass of the comet of the order of 10^{-6} of the Earth's mass.

Accurate measurement of the size of the nucleus is a matter of some difficulty, and various assumptions have always to be made in such work. The effect of phase, for instance, is not known, and is usually neglected, but there is no doubt that the actual dimensions of the nucleus vary over wide limits. Pons-Winnecke's comet in 1927 had a nucleus estimated at only 2 or 3 km. in diameter, while figures up to several thousands of kilometres have been given in other cases. A recent determination of the mass of Halley's comet (Vorontsov-Velyaminov, 1946) is based on measurements of the brightness of the Fraunhofer lines in the spectrum of the nucleus, and on deductions from the physical theory of spectra. A mass of 3×10^{19} gm. is thus found, which is of the same order as other estimates, but the diameter of the nucleus works out at 30 km., so that it must consist of a dense cluster of blocks of material whose distances apart are comparable with their dimensions. Other calculations have suggested that the nucleus consists of much smaller particles widely separated (Dubiago, 1942), but whichever theory is correct there is little doubt that the forces maintaining stability in a cluster of bodies of this kind are very remarkable, and it seems that gravitational attraction alone cannot account for this.

The mass of 3×10^{19} gm. mentioned above for Halley's comet is not small (approximately 3×10^{13} tons), but it is a very small fraction of the Earth's mass—about 5×10^{-9}. Many comets are much smaller than this, and hence the nucleus may be too small to be visible, or may be too scattered to form a definite image in the telescope. That the nucleus of a comet consists of solid matter in the form of a cluster of particles is supported also by spectroscopic evidence. The application of the slit spectroscope to the study of comets by Donati in 1864 was followed by the use of the objective prism by de la Baume Pluvinel in 1902. Both methods are in use to-day, and in the quartz slit spectrograph of the McDonald Observatory a dispersion of 69 A./mm. has been reached, enabling important advances to be made in the ultraviolet region.

Spectra

The spectrum of a comet consists essentially of a number of bands superimposed on a faint continuous background. In the brighter comets this continuous spectrum shows the usual Fraunhofer lines, so that it is clearly due to reflected sunlight. Some of the bands in the head of a comet were early identified as due to the molecules C_2, CH and CN, while N_2^+ and CO^+ were found in the tails. Recent work has succeeded in

identifying the remaining bands as due to the molecules OH, NH, CH^+, CH_2, NH_2 and OH^+. These molecules, which are chemically unstable, must owe their stability in the cometary atmosphere to the extremely low density, and the resultant freedom from collisions. In the case of a few comets which have made a sufficiently close approach to the Sun, the *D* lines of sodium appear in the spectrum, but this is the only emission line definitely identified in cometary spectra. The band spectra all involve the ground state of the molecules, and must be due to fluorescence excited by the Sun, a view which has been confirmed by McKellar (1942) in his work on the profiles of the CN bands. He showed that by making allowance for the profiles of the solar spectrum, the observed and calculated intensities for these bands were in good agreement.

The molecules which are found spectroscopically in the tails of comets are those which we should expect if the structure of a comet were similar to that of the meteorites which fall upon the earth. These bodies occlude gases which can be driven off by heating *in vacuo*, and the principal gases which have been found in this way are CO, CO_2, CH_4, H_2, N_2. Comets must also contain H_2O and NH_3, but otherwise the similarity between the two classes of body is remarkable. Remembering the close connection that is known to exist between comets and meteors, there is every reason for adopting the view that the nucleus of a comet consists of a cluster of meteoric particles, the comet being distinguished from a mere assemblage of such bodies by its compact nature, and by the presence of an atmosphere. It is generally agreed that this atmosphere has its origin in the gases which are occluded in the solid bodies, these gases being expelled by the heating effect of solar radiation as the comet approaches the Sun, and then repelled by radiation pressure and thus forming the tail of the comet.

Light Pressure

That light should exert a pressure of this kind was to be anticipated on Newton's corpuscular theory of the nature of light, and many unsuccessful attempts to detect the effect were made during the eighteenth and nineteenth centuries. Clerk Maxwell had given an expression for the effect based on his electromagnetic theory, and the cause of the failure to detect it was realised with the discovery of the radiometer by Crookes in 1892. The method of eliminating the radiometric effect was successfully achieved in 1900 by Lebedew in Russia, and independently by Nichols and Hull in America. A further refinement of the apparatus by Hull in 1906 enabled a measurement of the pressure to be made which agreed within 2 per cent of that expected on the basis of theory. Poynting and Barlow in 1904 were able to show the presence of the tangential component of this pressure, the reality of which was by this time beyond doubt.

COMET MOREHOUSE, 1908 III

Remarkable for the rapid variations in the structure of its tail. 1908 November 19.

(*Royal Observatory*, *Greenwich*)

Since the repulsive force due to radiation pressure will be proportional to the area of the body, while gravitational attraction varies with the mass, and therefore the volume, a point must eventually be reached when the two forces are equal. This will occur when the particle on which the radiation is falling has dimensions of the order of one micron ($\mu = 0{\cdot}001$ mm.) and Schwarzschild has shown that for smaller particles, radiation pressure exceeds gravitational attraction, and reaches a maximum for particles of dimensions one-third of the wave-length of the incident light. For particles of the size of gas molecules, very little repulsive effect is to be expected, since their dimensions are far too small. However, gases may absorb selectively the radiation falling upon them, and the pressure which may be produced in this way was demonstrated by Lebedew in 1906. It must not be concluded that the explanation of the formation of a comet's tail as due to radiation pressure (or selective absorption) is fully satisfactory. Eddington's work on Morehouse's comet of 1908 showed repulsive forces of the order of 800 times gravity, while regular changes took place in the appearance of the tail. Such a cycle of changes has been seen in other comets, while the formation of multiple tails, some of which are seen at a considerable angle from the line joining comet to Sun, also requires further investigation. Theories which have been used in the past to explain the formation of multiple tails as due to differing chemical composition cannot be maintained, since the spectrum of each individual tail is the same. The best explanation that can be given at present is that these tails are formed by some selective effect in which the particles of different sizes are repelled with different speeds.

Luminosity

Another unexplained feature in the behaviour of comets is the variation of luminosity. If a comet were to shine by reflected sunlight, its apparent luminosity would vary inversely with the square of its distance from the Earth and from the Sun. This would lead to an expression of the type

$$\frac{I}{I_0} = \frac{1}{r^2\,\Delta^2} \qquad \text{.....(1)}$$

where I is the luminosity at distance r from the Sun and Δ from the Earth, and I_0 is the standard luminosity, i.e., the luminosity at unit distance from both bodies. Since stellar magnitudes are related by the formula

$$\frac{I}{I_0} = k^{m_0 - m} \qquad \text{.....(2)}$$

where $\log k = 0{\cdot}4$,
we may combine the two laws in the logarithmic form

$$m - m_0 = 5 \log r + 5 \log \Delta \qquad \text{.....(3)}$$

It is found in practice that very few comets obey such a simple law, and observers generally are able to deduce some other law (a different one for each comet) by a least square solution of a large number of observations. Thus it is common to find a fourth or a sixth power law used, i.e., formulae of the type

$$\frac{I}{I_0} = \frac{1}{r^4 \Delta^2} \quad \text{or} \quad \frac{I}{I_0} = \frac{1}{r^6 \Delta^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Unfortunately it is not a simple matter to estimate the magnitude of a comet, since it does not form a point-source of light like a star, and observers differ widely in their methods of dealing with this problem. If we take the observations of magnitude of one observer, however, we can visualise the changes that take place in the brightness of a comet. In Fig. 3 the magnitudes of Comet de Kock-Paraskevopoulos 1941 IV as observed by Wood are plotted against the time of observation. Using an equation of the form

$$m - 5 \log \Delta = H_0 + y \log r$$

Wood (1941) finds the standard magnitude $H_0 = 6{\cdot}91$ and the constant $y = 12{\cdot}3$, from which the law of luminosity is seen to follow almost a fifth power of r. The curve representing this equation is shown as a continuous line in the figure.

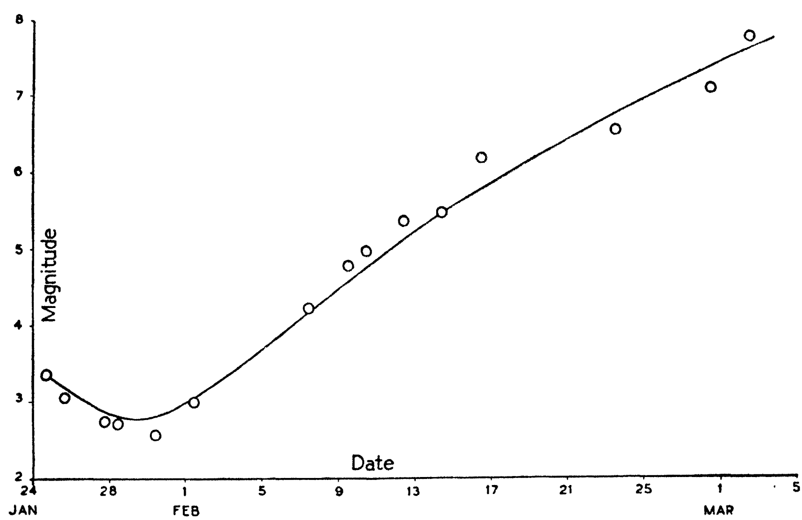


FIG. 3.—Light curve for Comet 1941 IV.

The agreement in this case is unusually close, for it is not uncommon for a comet to show unexpected outbursts of light, or to fade, after perihelion passage, more rapidly than would be predicted from such empirical formulae. In some cases there would appear to be a period of

some days in which the comet loses or gains in brightness, but here again the observations, separated as they are by an interval of at least a day, are not sufficiently comprehensive to confirm the presence of any such periodicities. Many types of formulae have been proposed to deal with the variations in the brightness of comets, but until something more is known of the causes of these changes, little progress is likely to be made. The sudden changes in the light of Comet Morehouse (1908 III) led at the time to the suggestion that there was a correlation between these outbursts and the appearance of sunspots. The disruption of the tail of Whipple-Fedke-Tevzadze 1943 I in March 1943, occurred within a few hours of the appearance of a fine aurora, and there is a strong presumption of some connection between the two events.

The most remarkable changes of this kind are observed in P/Schwassmann-Wachmann 1925 I. On 1946 January 1 it was of the 16th magnitude, but it had risen to magnitude 10·2 on January 25, and 9·4 on January 26. The outburst did not last long, for by February 8 the brightness had dropped to magnitude 15. Nicholson (1947) called attention to the presence at the time of a giant sunspot, turned towards the comet, and suggested the possibility of further outbursts during that year of maximum solar activity. The comet brightened again by some five magnitudes during November 1947, but it is difficult to appreciate how the Sun could influence this comet, at a distance of some six or seven units, to a sudden increase of its light by a factor of 1,000 or more. Moreover, comets such as Biela or Brooks, which have split into two or more parts, have shown variations in the brightness of one part only, and a solar influence would seem to be out of the question in such cases.

Attempts have been made, particularly by Vsessviatsky (1933–1937) to determine the absolute magnitude of comets, that is the magnitude m_0 in equation (3), which represents the apparent brightness of the comet at unit distance from the Sun and from the Earth. Such values are, however, based on a number (usually quite small) of discordant estimates which are affected by large systematic errors. Vsessviatsky's figures can only be regarded as statistical averages, and results derived from them, such as the reputed fading of Encke's comet, must be regarded as doubtful. Some recent work by Bobrovnikoff (1942) promises to give more accurate data on the basis of a reduction of the original data to a common photometric system.

Whipple's hypothesis

Many of the difficulties which occur in the behaviour of comets, and which have not hitherto found an adequate explanation, are overcome in the comet model which Whipple (1950) has suggested. He imagines the

nucleus of a comet to consist of a conglomerate of meteoric materials with " ices " consisting of gases such as H_2O, NH_3, CH_4, CO_2, solidified under the lower temperatures of interplanetary space. Solar radiation will vaporise some of the gases, leaving an outer matrix of poorly conducting material ; and a mathematical treatment of the conditions then prevailing shows that heat transfer, which takes place through the thin meteoric layers mainly by radiation, is subject to a considerable time-lag if the nucleus is rotating.

Solar radiation strikes the comet at an angle to its direction of motion, and if the nucleus is rotating in the same sense as the motion round the Sun, then the time-lag will result in the emission of vapours from the nucleus with a component in the reverse direction. The resultant increase in the velocity of the comet will cause an increase in the semi-major axis of the orbit, which will give an apparent decrease in the mean motion (see p. 54). Examples of this decrease are to be found in Comets d'Arrest and Wolf I. The hitherto unexplained acceleration in the motion of Comet Encke is to be explained on this hypothesis if the nucleus of this comet is rotating in a retrograde sense, the loss of mass required to account for the effect being computed as only 0·2 per cent per revolution. If this attractive idea can be substantiated by experimental results, a notable advance will have been made in cometary dynamics.

Orbit computing

The mathematical study of cometary orbits has already been responsible for a literature so vast as to be out of all proportion to the results achieved. It would be true to say that this great volume of work has done little but improve the methods laid down by Laplace, Gauss, and others a century and a half ago. The main improvements have arisen as a direct result of the use of calculating machines, yet the larger problems of the dynamics of cometary systems remain unsolved. The advent of the electronic calculating machine would seem to presage a new era in such research, for these machines are capable of solving, by purely arithmetical processes, problems which are beyond the reach of pure mathematics. The beauty of the old classical mechanics remains, but to-day it is beginning to be realised that, in some directions at least, there are other and better methods.

BIBLIOGRAPHY

A. BOOKS

CHAMBERS, G. F., 1909, *The Story of the Comets* (Oxford : Clarendon).

CLERKE, A. M., 1895, *History of Astronomy during the Nineteenth Century* (Edinburgh : A. & C. Black).

CROMMELIN, A. C. D., and PROCTOR, M., 1937, *Comets* (London : Technical Press).

HIND, J. R., 1852, *The Comets* (London : Parker).

OLIVIER, C. P., 1930, *Comets* (London : Baillière, Tindall & Cox).

WATSON, FLETCHER, 1941, *Between the Planets* (Philadelphia : Blakiston).

WILLIAMS, J., 1871, *Chinese Observations of Comets* (London).

B. REPORTS AND LECTURES

CROMMELIN, A. C. D., 1929, Article *Comets* in *Encyclopædia Britannica*, 14th Edition.

HOGG, F. S., 1929, *A Synopsis of Cometary Spectra*, *J.R.A.S.Canada*, **23**, 55, = *Harvard Reprint*, 57.

MERTON, G., 1951, *Comets*, *Brit. Astr. Assocn. J.*, **62**, 4.

PEARSE, R. W. B., 1948, *Spectroscopic Observations on Comets, etc.*, *Phys. Soc. Rep. Prog. Phys.*, **11**, 410.

SWINGS, P., 1943, *Cometary Spectra*, *Mon. Not. Roy. Astr. Soc.*, **103**, 86.

TURNER, H. H., 1910, *Halley's Comet* (Oxford : Clarendon). (*The Halley Lecture*, 1908.)

VINTER HANSEN, J., 1944, *The Physical Structure of Comets*, *Pub. Astr. Soc. Pac.*, **56**, 182.

C. REFERENCES

BOBROVNIKOFF, N. T., 1942, *Pop. Astr.*, **50**, 473.

DUBIAGO, A. D., 1942, *Russ. Astr. J.*, **19**, 14.

McKELLAR, A., 1942, *Rev. Mod. Phys.*, **14**, 179

NICHOLSON, S. B., 1947, *Pub. Astr. Soc. Pac.*, **59**, 30.

VORONTSOV-VELYAMINOV, B., 1946, *Astrophys. J.*, **104**, 226.

VSESSVIATSKY, S., 1933, *Russ. Astr. J.*, **10**, 327. (In English.)

VSESSVIATSKY, S., 1935, *Russ. Astr. J.*, **12**, 243. (In English.)

VSESSVIATSKY, S., 1937, *Russ. Astr. J.*, **14**, 480. (In English.)

WHIPPLE, F., 1950, *Astrophys. J.*, **111**, 375, and **113**, 464.

WOOD, H. W., 1941, *Brit. Astr. Assocn. J.*, **51**, 284.

CHAPTER II

The Orbit of a Comet

The movements of the planets, which had for so long attracted the attention of mankind, were shown by Johann Kepler in the early part of the seventeenth century to be governed by certain laws, of which three are always associated with his name :

1. *The orbit of a planet is an ellipse, with the Sun at one of its foci.*
2. *The line joining the planet to the Sun sweeps out equal areas in equal times.*
3. *The squares of the periods of the planets are proportional to the cubes of their mean distances from the Sun.*

These laws of Kepler were evolved as the result of immense labour in comparing the observations of Tycho Brahe, particularly those of the planet Mars, with every possible hypothesis as to the type of motion of the planets. The laws, as they stood, were thus purely empirical, and of an accuracy which matched that of the observations which had been amassed by Tycho. More accurate measurements would have defeated their purpose, and Kepler would doubtless have failed to establish these laws. Yet they may be deduced from, and are thus a natural consequence of, the universal law of gravitation, although in point of fact Newton demonstrated the truth of his law by a consideration of Kepler's Laws, of the motion of the Moon round the Earth, and of the gravity at the Earth's surface.

The first of Kepler's Laws may be stated in modern terms thus : *The undisturbed orbit of a body moving under the influence of the Sun's attraction is one of the conic sections, with the Sun in one focus.* In the case of an elliptical or hyperbolic orbit in space, six quantities or *elements* are needed to define the orbit completely. In the case of the parabola, in which the eccentricity is unity, only five elements have to be determined; and as this greatly simplifies the computation, the preliminary orbit of a newly discovered comet is always calculated as a parabola. The resulting orbit and the ephemeris derived from it, are generally sufficiently accurate to allow the object to be followed until enough data accumulate to enable a more accurate orbit to be determined. Even so, the average comet is seen for so short a time, and therefore over only a short arc of its orbit, that the exact determination of the elements is a matter of some difficulty.

In particular, the curvature of an ellipse or hyperbola with eccentricity near to unity, is practically indistinguishable from that of a parabola with the same perihelion distance. In such cases, the parabolic orbit is often sufficiently accurate, and hence about half of the orbits given in the catalogues of comets are parabolic.

The elements of the orbit

In Fig. 4, PBA is an ellipse with centre C and major axis PA. The Sun is assumed to be at the focus S.

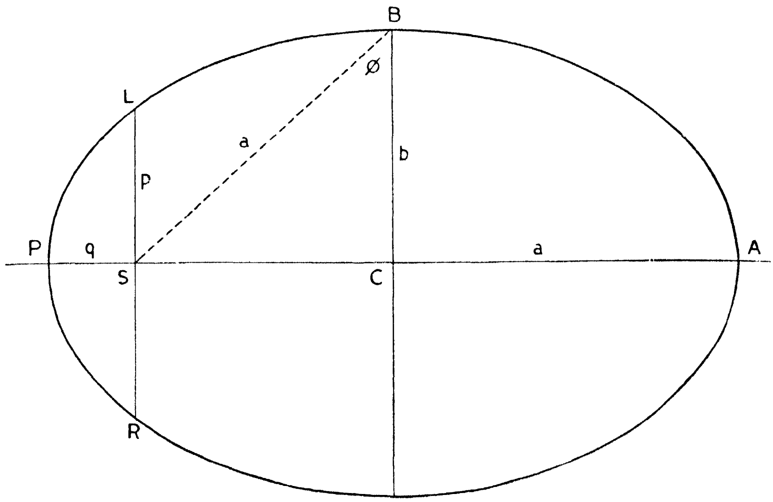


FIG. 4.—The elliptical orbit.

a is the *semi-major axis* CP, and this quantity defines the *size* of the orbit. The *semi-minor axis* CB is represented by the symbol *b*.

e is the *eccentricity* of the ellipse, and may be defined as the ratio CS : CP. It follows that CS $= ae$ and SP $= a(1 - e)$.

A comet at P is nearest the Sun, and is said to be at *perihelion*, while at A, the comet is at *aphelion*, and is then farthest from the Sun. The line PA is sometimes referred to as the *line of apsides*.

q is the *perihelion distance* (SP) and is equal to $a(1 - e)$.

q′ is sometimes used for the *aphelion distance* (SA) which is equal to $a(1 + e)$.

The eccentricity may be expressed in other ways, for by the properties of the ellipse, CP $=$ SB $= a$. Hence *e* is also the ratio CS : SB, and may thus be regarded as the sine of the angle SBC, the *angle of eccentricity* ϕ. Hence $e = \sin \phi$, an expression which is useful

in computing work, since it leads to $b = a \cos \phi$. It follows that $b^2 = a^2(1 - e^2)$.

p is the *semi-latus rectum* SL and is given by $p = a(1 - e^2) = \dfrac{b^2}{a}$

The above symbols are used to define an elliptical orbit, but may be used for a hyperbola, in which e is greater than 1, by making a negative. In the case of a parabola, however, in which e is unity, the symbols a and b no longer have a real meaning, and in this case, the shape and size of the orbit are defined completely by the symbol q. The semi-latus rectum in this case is $2q$. The remaining symbols are the same for all types of orbit.

i (see Fig. 5) is the *inclination* of the plane of the comet's orbit to the plane of the ecliptic. It may have any value from 0 to 180°, and values greater than 90° indicate that the comet is travelling round the Sun in the opposite direction to that of the planets, i.e., its motion is retrograde.

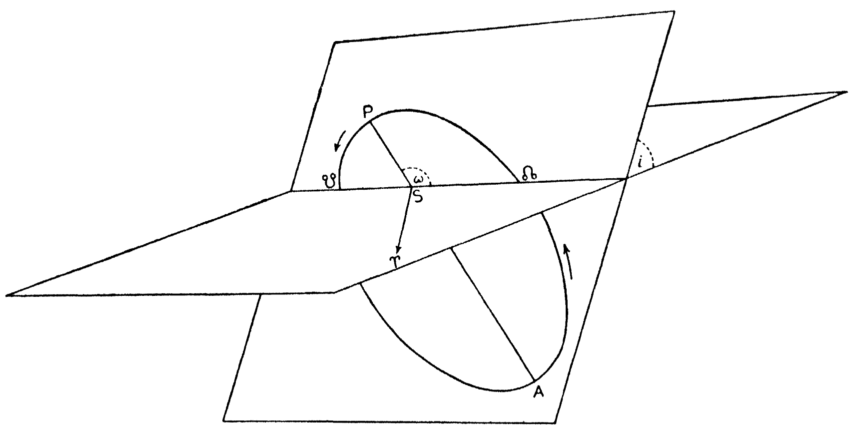


FIG. 5.—The plane of the orbit.

☊ is the *ascending node*—the point at which the comet passes from the south to the north side of the ecliptic. The symbol is more generally used to represent the longitude of this node, i.e., the angle ♈S☊ measured from the vernal equinox ♈ in the plane of the ecliptic. The symbol ☋ is used in the same way with reference to the descending node, whose longitude will clearly be ☊ ± 180°.

ω is the *argument of perihelion*, and is the angle ☊SP, *measured in the plane of the comet's orbit*. The older method of quoting the *longitude of perihelion* $\pi =$ ☊ $+ \omega$ is not now used in cometary work.

It will be seen that the size of the orbit is defined by a, its shape by e, and the orientation of its plane in space by i and Ω, while the direction of the major axis within that plane is given by ω. In order to define the position of the comet in the orbit, it is necessary to add a sixth element, which is usually the time of *perihelion passage* T. If this is not done, then it is necessary to give the position of the comet at some other *epoch*. The position is defined by means of angles which are always known as *anomalies*. In Fig. 6, the position of the comet is given by M, and the quantities that may be considered as determining the position in the orbit are here defined, together with certain auxiliary quantities which are of value in computing the position.

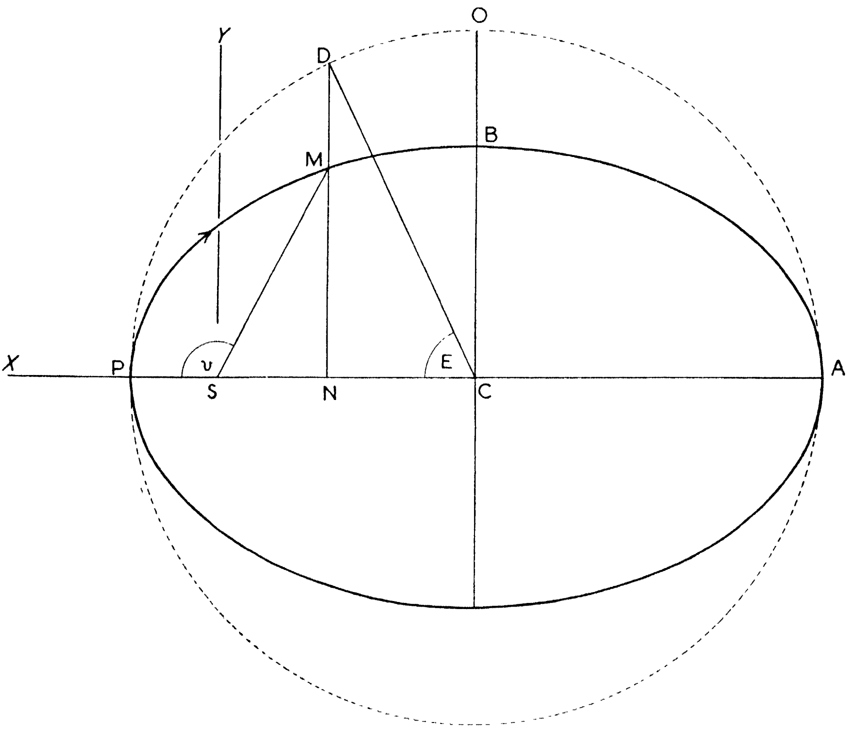


FIG. 6.—True and eccentric anomalies.

r is the *radius vector* SM.

v is the *true anomaly* and is the angle between the radius vector and the line of apsides, i.e., PSM in Fig. 6. The true anomaly is always measured in the direction of motion of the comet.

P is the *period* of revolution of the comet.

n is the *mean daily motion*, generally expressed in degrees, although other units are sometimes employed.

E is the *eccentric anomaly*, PCD, the point D being on the auxiliary circle AOP, and DN being drawn through M perpendicular to the major axis.

M is the *mean anomaly*, defined by $n(t - T)$ where t is the date and T the time of perihelion passage. M may be regarded as the angle through which the radius vector would turn if the comet moved in a circular orbit with the same period. It is to be noticed that M and E are zero when v is zero, and all three are also coincident at 180°.

The position of the comet in the orbit may then be defined by reference to the angles v or E. It is customary in modern practice to measure these angles in degrees and decimals (rather than in sexagesimal measure), while the various distances, such as r, a or q, are understood to be in astronomical units (1 a.u. = 93 million miles).

The equations of motion

If the Sun is taken as the origin of X- and Y-axes in the plane of the orbit, the X-axis being directed towards the perihelion (see Fig. 6), then the rectangular co-ordinates x, y, are related to the polar co-ordinates r, v by the equations

$$\begin{aligned} x &= r \cos v \\ y &= r \sin v \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5) \end{aligned}$$

Now the comet describes its orbit under the influence of the central force, which is the gravitational attraction of the Sun, and this will cause an acceleration along the radius vector. The components of this acceleration along the two axes will be

$$\begin{aligned} \frac{d^2x}{dt^2} &= -A \cos v = -Ax/r \\ \frac{d^2y}{dt^2} &= -A \sin v = -Ay/r \qquad \ldots\ldots\ldots\ldots\ldots\ldots (6) \end{aligned}$$

the negative sign indicating that the force is directed towards the origin. Multiplying these equations by y and x respectively, and subtracting,

$$x \frac{d^2y}{dt^2} - y \frac{d^2x}{dt^2} = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

the integral of which is

$$x \frac{dy}{dt} - y \frac{dx}{dt} = h \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

where h is the constant of integration. This equation becomes, in polar co-ordinates, using (5),

$$r^2 \frac{dv}{dt} = h \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which the left hand side is seen to be twice the area of the sector swept out by the radius vector in unit time. This equation is therefore a mathematical statement of Kepler's second law, which is clearly a consequence of the fact that the force of attraction is a central one.

Multiplying equations (6) by x/r and y/r respectively, and adding,

$$\frac{x}{r} \cdot \frac{d^2x}{dt^2} + \frac{y}{r} \cdot \frac{d^2y}{dt^2} = -A \qquad \text{(10)}$$

since $x^2 + y^2 = r^2$. Equation (10) may be expressed in polar co-ordinates in the form

$$\frac{d^2r}{dt^2} - r\left(\frac{dv}{dt}\right)^2 = -A \qquad \text{(11)}$$

a result which also follows from successive differentiation of (5). Now the polar equation of a conic with axes as in Fig. 6, is

$$r = p/(1 + e \cos v) \qquad \text{(12)}$$

and this, on differentiation, gives, with the aid of (9),

$$\frac{dr}{dt} = \frac{eh \sin v}{p} \qquad \text{(13)}$$

$$\frac{d^2r}{dt^2} = \frac{eh^2 \cos v}{r^2p} \qquad \text{(14)}$$

Hence, from (11), and using (9) again,

$$A = \frac{h^2}{r^3} - \frac{eh^2 \cos v}{r^2p} = \frac{h^2}{pr^2} \qquad \text{(15)}$$

Since h and p are constant, the acceleration, and therefore the force that causes it, is seen to be inversely proportional to the square of the distance. The law of force being known, it is then possible to establish certain important relations in connection with the comet's movements.

The motion of the comet

According to Newton's law of gravitation, the force of attraction between two bodies of masses M and m, separated by a distance r is given by

$$F = GMm/r^2 \qquad \text{(16)}$$

where G is the gravitational constant. The force of attraction will be the same on each body, but the resultant accelerations will be different. Since force is measured by the product of mass and acceleration, the smaller body of mass m will have an acceleration GM/r^2 towards the larger body, while the latter will have an acceleration of Gm/r^2 towards the first. The total acceleration of the smaller body relative to the larger will thus be $G(M + m)/r^2$, and if μ is the acceleration at unit distance, $\mu = G(M + m)$. It is usual in astronomical work, in which the square root of G is often

required, to substitute $G = k^2$, and to measure the masses in terms of the Sun's mass as unity. Thus

$$\mu = G(1 + m) = k^2(1 + m) \quad \ldots\ldots(17)$$

and it is important to notice that although G and k are constants, the quantity μ varies with the mass of the body. Since the mass of a comet is negligible, it is sufficiently accurate in this case to write $\mu = G = k^2$.

The acceleration given by (15) may now be written

$$A = h^2/r^2p = \mu/r^2 \quad \ldots\ldots(18)$$

from which

$$\mu = h^2/p \quad \ldots\ldots(19)$$

and in the case of an elliptical orbit the value of this quantity may be obtained without difficulty. Since h is twice the area swept out by the radius vector in unit time,

$$h = \frac{\textit{twice the area of the ellipse}}{\textit{period}} = \frac{2\pi ab}{P} \quad \ldots\ldots(20)$$

In this expression, $2\pi/P$ is the mean motion, although it is here given in radians instead of the more usual degrees per unit of time. Putting $n = 2\pi/P$, it follows that

$$h = nab \quad \ldots\ldots(21)$$

From equations (19) and (21) it is then easy to deduce that

$$\mu = \frac{h^2}{p} = n^2a^3 \quad \ldots\ldots(22)$$

and on combining this result with (17),

$$n^2a^3 = k^2(1 + m) \quad \ldots\ldots(23)$$

Thus for two different bodies of masses m_1 and m_2, there exist the relations

$$\begin{aligned} n_1^2a_1^3 &= k^2(1 + m_1) \\ n_2^2a_2^3 &= k^2(1 + m_2) \end{aligned} \quad \ldots\ldots(24)$$

from which by division,

$$\frac{a_1^3}{a_2^3} = \frac{(1 + m_1)}{(1 + m_2)} \cdot \frac{P_1^2}{P_2^2} \quad \ldots\ldots(25)$$

Kepler's third law is an approximate form of (25) obtained by neglecting the masses m_1 and m_2.

Equation (23), which is of particular importance in dealing with the motion of comets or planets, may be written

$$P = 2\pi a^{\frac{3}{2}}/k(1 + m)^{\frac{1}{2}} \quad \ldots\ldots(26)$$

and was used in this form by Gauss to determine the value of k. In the case of the earth, $a = 1$, so that

$$P = 2\pi/k(1 + m)^{\frac{1}{2}} \quad \ldots\ldots(27)$$

and using the values $P = 365{\cdot}2563835$ mean solar days

$m = 1/354710$

this expression gives $k = 0{\cdot}01720\ 209895$. Modern values of P and m differ from those used by Gauss, but it is more convenient to retain his value of k, and adjust the values of the unit of distance so that equations (22) and (23) always hold. If the earth's mass were negligible, the length of the year P_0 would be $2\pi/k = 365{\cdot}256898$ mean solar days, or $1{\cdot}00004027$ tropical years of $365{\cdot}24219$ days. Since the mass of a comet is also negligible, its period P will be given by

$$P = P_0 \,.\, a^{\frac{3}{2}}$$
$$= a^{\frac{3}{2}} \quad \text{" Gaussian " years of } 365{\cdot}256898 \text{ days.} \qquad \ldots\ldots(28)$$

The period of a comet, when given, is usually expressed in these " Gaussian " years, but is of little value except to give a rough indication of the time of the next return of a periodic comet. Of more use to the computer is the relationship derived from (23),

$$n = ka^{-\frac{3}{2}} \qquad \ldots\ldots(29)$$

in which the mean motion n may be expressed in various units through the following values of k :

$$k = 0{\cdot}01720\ 209895 \text{ astronomical units per day}$$
$$= 0{\cdot}98560\ 76686 \text{ degrees per day}$$
$$= 29{\cdot}80 \text{ kilometres per second.}$$

The astronomical unit referred to is, of course, the special Gaussian unit, i.e., the radius of a circular orbit in which a body of negligible mass would revolve about the Sun in P_0 days.

The value of k also enters into the expression for the velocity of the comet in its orbit, which may be derived from

$$V^2 = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$$

This may be written in polar co-ordinates in the form

$$V^2 = r^2\left(\frac{dv}{dt}\right)^2 + \left(\frac{dr}{dt}\right)^2$$
$$= \frac{h^2}{r^2} + \frac{e^2\,h^2 \sin^2 v}{p^2} \qquad \text{(using (9) and (13))}$$
$$= \frac{h^2}{p^2}(1 + e^2 + 2e \cos v)$$
$$= k^2(2/r - 1/a) \qquad \ldots\ldots(30)$$

This formula is important since it gives the velocity at any point in the orbit. If the body is moving in a parabola, then $a = \infty$ and $V^2 = 2k^2/r$. This is the velocity which the body would acquire in falling freely to the given point from infinity ; and if the body is projected with this velocity (at the distance r from the Sun), then it will describe a parabolic orbit, no matter what the initial direction of its motion. If the velocity is less than this parabolic value, then a must be positive, and the orbit is an ellipse ;

while a velocity greater than parabolic is obtained if a is negative, the orbit then being hyperbolic. This is indicated more clearly if the parabolic velocity is represented by U, so that

$$U^2 = 2k^2/r \qquad (31)$$

and the constant k is then eliminated between (30) and (31), giving

$$a = \frac{r}{2}\left(\frac{U^2}{U^2 - V^2}\right) \qquad (32)$$

from which it is clear that a is positive, infinite or negative, according as V is less than, equal to, or greater than U. Further, the expression contains only measurements of velocity and distance, and the direction of projection does not enter into the matter at all. This direction merely controls the orientation and shape of the resulting orbit ; thus in Fig. 7, the two ellipses have the same velocities at the point P, and the same values of a and of n.

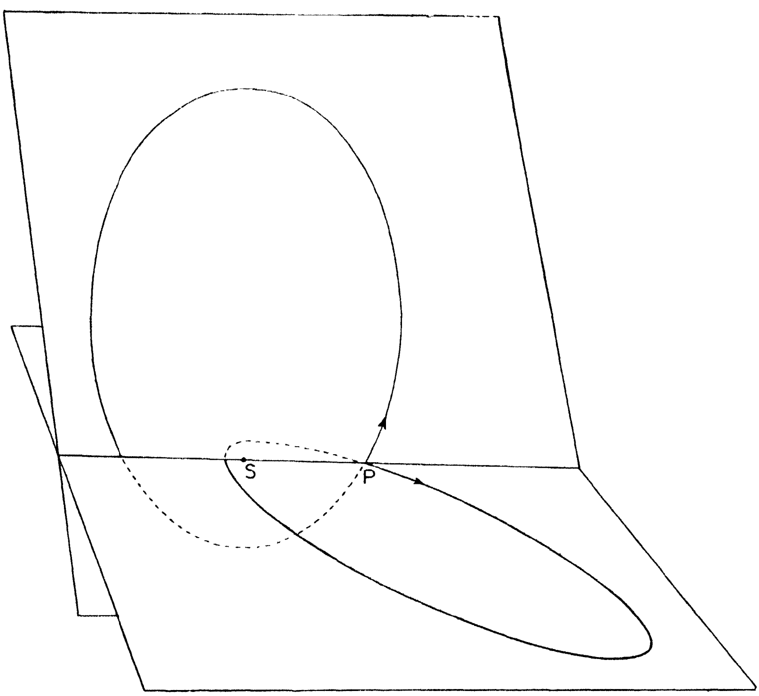


FIG. 7.—Different ellipses having the same velocity of projection.

If d is the length of the perpendicular from the Sun on to the tangent, then the area of the sector described in unit time may be written as $\frac{1}{2}Vd$, so that, using (19),

$$Vd = h = k\sqrt{p} \qquad (33)$$

If the tangent makes an angle σ with the radius vector, $d = r \sin \sigma$ so that

$$Vr \sin \sigma = k\sqrt{p} \qquad (34)$$

and it is seen that the direction of projection determines the value of the parameter p, and hence of the eccentricity of the orbit.

Since in equation (30) the mass of the comet has been neglected, k^2 having been used instead of the more accurate μ, it is clear that two bodies travelling in the same orbit will have the same velocity at a given point only if they have identical masses. In all other cases the velocities will differ, however slightly, and the two bodies will tend to separate in the orbit. In the case of an ellipse the same fact is seen to be implied in equation (26) for the time of revolution of the body about the Sun.

Position in the orbit

Although the position of the comet in the orbit may be defined by means of the true anomaly v, it is more convenient in actual practice to use the eccentric anomaly E. Since by the properties of an ellipse and its auxiliary circle (Fig. 6),

$$MN/DN = b/a$$

it follows that

$$r \sin v = b \sin E$$

$$r \cos v = a(\cos E - e) \qquad (35)$$

Squaring and adding these equations, with some necessary rearrangement,

$$r = a(1 - e \cos E) \qquad (36)$$

which is an alternative and more useful form of (12). Differentiating this equation and using (13),

$$\frac{dr}{dt} = ae \sin E \, dE/dt \qquad (37)$$

$$= \frac{eh \sin v}{p}$$

so that

$$\frac{dE}{dt} = \frac{h \sin v}{pa \sin E} = \frac{h}{rb} = \frac{na}{r} \qquad (38)$$

whence

$$n = \frac{r}{a} \cdot \frac{dE}{dt} = (1 - e \cos E) \, dE/dt \qquad (39)$$

the integral of which is

$$n(t - T) = E - e \sin E \qquad (40)$$

if E is zero at time T. The left-hand side of this expression is the mean anomaly M, so that

$$M = E - e \sin E \qquad (41)$$

This very important relation between M and E is known as *Kepler's Equation* and must be solved to find either E or v.

The equation is transcendental and is solved by successive approximations. Some hundreds of methods have been published for obtaining close approximations to the correct value of E, and there is an extensive literature on the subject (see Wood, 1950). With a hand calculating machine, however, solution of this equation presents no difficulties. In order to work in degrees, the equation is transformed by writing

$$e^\circ = 57{\cdot}29578\, e$$

and computing from the form

$$E = M + e^\circ \sin E.$$

The value of $M = n(t - T)$ for the required date is set in the product register of the machine, and e° on the setting levers ; then, using suitable tables in degrees and decimals, the handle is turned until the product register shows the value of E whose sine is given in the multiplier register.

An alternative process having many advantages has been suggested by B. Strömgren. The equation is written in the form

$$\sin E = E/e - M/e$$

and in this case, the setting levers contain the reciprocal of e, while $- M/e$ is set in the product register. This is a building-up process, the value of E appearing step by step in the multiplier register, and its sine in the product register.

The accuracy with which the result is obtained is controlled entirely by the factor

$$\frac{1}{1 - e \cos E}$$

which becomes large when e is greater than, say, 0·9 and E is small. Since most search ephemerides are required in the region of perihelion ($E = 0$), the calculation in the case of a nearly-parabolic orbit is always troublesome. A convenient method of overcoming the difficulties in such a case (elliptical or hyperbolic orbits having e near to unity) is given by Møller (1933), and where greater accuracy is required, the standard textbooks may be consulted. In the case of a parabolic orbit, Kepler's equation no longer applies, and the true anomaly must always be found in this case. Since in the parabola

$$r^2 \frac{dv}{dt} = k\sqrt{p} = k\sqrt{2q} \qquad \text{(42)}$$

and

$$r = 2q/(1 + \cos v) = q \sec^2 \frac{v}{2} \qquad \text{(43)}$$

the integral of (42) becomes

$$\frac{k(t - T)}{\sqrt{2}\, q^{\frac{3}{2}}} = \tan \frac{v}{2} + \frac{1}{3} \tan^3 \frac{v}{2} \qquad \text{(44)}$$

which is a cubic equation in $\tan v/2$, having only one real solution, which is positive for $v < 180°$. Tables giving the solution of this equation (known as Barker's Tables) may be found in the standard books, but for ephemeris purposes, the simple tables prepared by Strömgren (1927) and Møller (1932) are to be preferred.

The true anomaly in an elliptical orbit may be obtained from various tables (Astrand, 1890 ; Schlesinger and Udick, 1912) although these are usually of limited application. The value of v may be computed from (35), but a more convenient expression may be derived from the second of these equations and (36), in terms of the half angles :—

$$r \sin^2 \frac{v}{2} = a(1 + e) \sin^2 E/2$$

$$r \cos^2 \frac{v}{2} = a(1 - e) \cos^2 E/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (45)$$

from which

$$\tan \frac{v}{2} = \left(\frac{1 + e}{1 - e}\right)^{\frac{1}{2}} \tan E/2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (46)$$

Since the value of E is readily obtained with a small machine, this formula enables the value of v to be determined quickly and accurately in cases of elliptical orbits of moderate eccentricity.

The empty focus

For the purpose of providing a rough picture of the conditions at a given time, it is often sufficiently accurate to assume that the angular velocity of the body about the empty focus is uniform. In Fig. 8 the

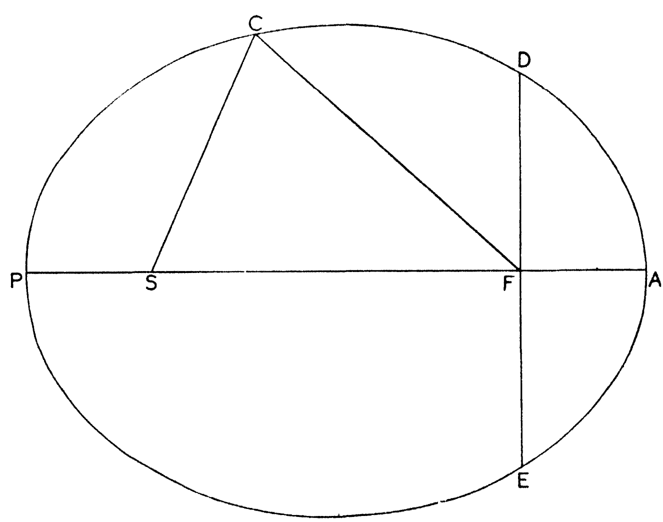


FIG. 8.—The empty focus.

angular velocity of the comet C about the Sun S is dv/dt and this varies with the position in the orbit. If F is the empty focus, then the lines SC and FC are equally inclined to the tangent at C, so that the angular velocities about the two foci are inversely proportional to the distances. Thus if dw/dt is the angular velocity about F, then

$$r \,.\, dv/dt = (2a - r) .\, dw/dt \qquad (47)$$

from which, using (9),

$$\frac{dw}{dt} = \frac{h}{r(2a - r)} = \frac{nab}{r(2a - r)} \qquad (48)$$

This expression has its maximum value of $n.a/b$ when the comet is at the ends of the major axis, and its minimum value of $n.b/a$ when the comet is at the ends of the minor axis, so that in the case of an ellipse of small eccentricity, the angular velocity about F approximates to n. Even in the case of an elliptical orbit of moderate eccentricity the expression is still close enough to the truth to be useful. In Fig. 9 the relative values of the angular velocities dM/dt, dE/dt, dv/dt and dw/dt are shown for various positions of a comet in an elliptical orbit of eccentricity $1/\sqrt{2}$. These values have been chosen as representing the ordinary type of short-period comet, and the graphs show clearly the manner in which these angles change as the comet proceeds round the orbit.

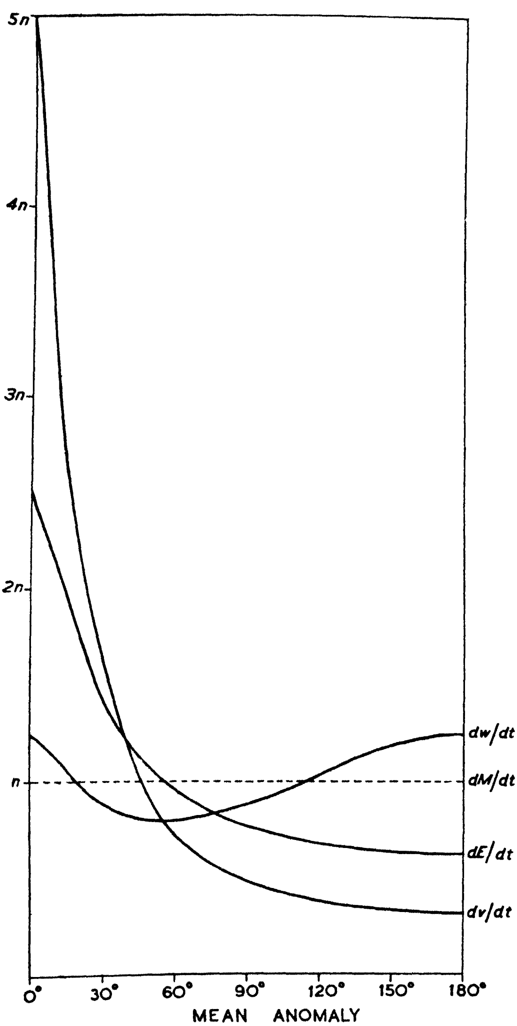


FIG. 9.—Graphs of angular velocities.

Perhaps the most interesting feature of this simple approximation is that it brings out very clearly the fact that a comet spends half of its period in the

portion DAE of its orbit (Fig. 8). Now the distance FD is simply p, the semi-latus rectum, so that SD $= 2a - p = a(1 + e^2)$. In the case of the short-period comet considered in Fig. 8, this distance is about 5·4 units. Since the mean distance of Jupiter from the Sun is 5·2 units, it is seen that these short period comets can make quite close approaches to that planet. Moreover, the velocity of the comet at the point D is given by putting $r = 5{\cdot}4$ in (30), and for this representative comet, a value of $0{\cdot}35k$ is obtained. The mean velocity of Jupiter is about $0{\cdot}43\ k$ in the same units, so that comet and planet, moving in the same direction, have speeds of the same order of magnitude and the comet will remain for some time in the neighbourhood of Jupiter. It is this prolonged influence of Jupiter on the short period comets at aphelion that is responsible for the changes that take place in their orbital elements.

In the case of an elliptical orbit of great eccentricity, the empty focus rule cannot give good results. In Fig. 10, the computed positions of

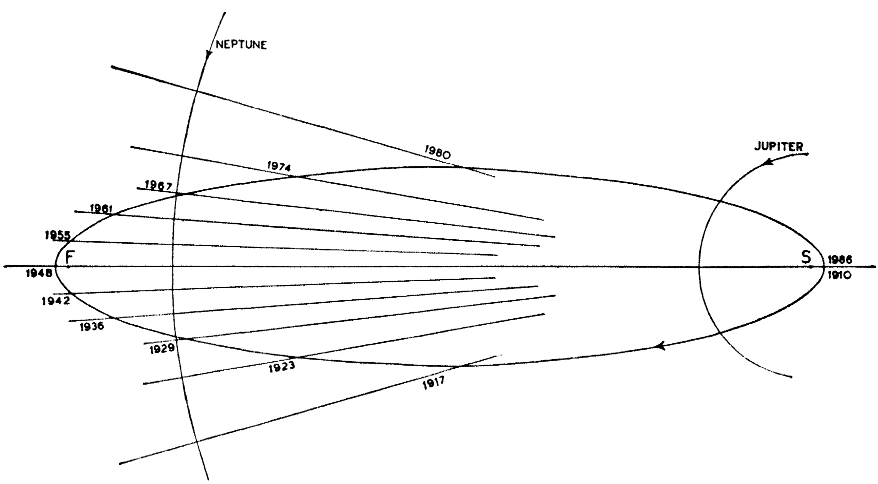


FIG. 10.—Halley's comet and the empty focus.

Halley's comet are shown at twelve equal intervals of time, and it will be seen that the motion about the empty focus is far from uniform. The rapid changes in the true anomaly are such that when v is 120^0, the value of M is still only $1^{\circ}{\cdot}6$. The rapid motion near the Sun, and very slow motion near aphelion, result in Halley's comet spending more than half of its period of 75 years beyond the orbit of Neptune.

The orbit in space

The position of the comet has so far been discussed with reference to the ellipse in which it travels, and for this purpose only two dimensions

are required, with X- and Y-axes in the plane of the orbit. In order to obtain the position of the comet as it is seen from the earth, it is necessary to use three dimensions. Suppose the position C of the comet to be referred in the first instance to three axes, P, Q and R, centred at the Sun, of which P is directed to the perihelion of the orbit, Q to a point on the orbit where the true anomaly v if 90°, and R to the pole of the orbit. The axis R is thus normal to the orbit plane in which the other two axes lie, and any ambiguity is removed by the convention that an observer at any point on the R- axis (drawn from the Sun) would see the comet moving in a counter-clockwise direction. The co-ordinates of the comet referred to this system are then

$$r \cos v, \qquad r \sin v, \qquad 0.$$

The point C is now transferred to the more usual system of axes based on the Earth's equator. These axes are X, Y, Z, and are again centred at the Sun ; X is directed to the vernal equinox, Y to a point on the equator in right ascension 6 hours (i.e., 90° in advance of the vernal equinox), and Z to the north pole. Then the co-ordinates of C in this equatorial system may be obtained by projection, the value of the x co-ordinate, for example, being the sum of the projections of the original co-ordinates on the new X-axis. If P_x, P_y, P_z are the direction cosines of the point P with reference to the axes X, Y, Z, and Q_x, Q_y, Q_z and R_x, R_y, R_z similarly refer to Q and R, then the heliocentric equatorial co-ordinates of C are given by

$$\begin{aligned} x &= P_x . r \cos v + Q_x . r \sin v \\ y &= P_y . r \cos v + Q_y . r \sin v \\ z &= P_z . r \cos v + Q_z . r \sin v \qquad \ldots\ldots\ldots\ldots (49) \end{aligned}$$

It will be noticed that the R's do not appear in these equations, since the original R co-ordinate was zero. Nevertheless, the R's are of value in certain classes of orbit work, and they are discussed in some detail by Smiley (1930). The P's, Q's and R's have all the usual properties of direction cosines, and in particular $\Sigma P^2 = \Sigma Q^2 = \Sigma R^2 = 1$, while the actual coefficients of equation (49) are readily checked, both as to magnitude and sign, by using the orthogonal relationship $\Sigma PQ = 0$.

With co-ordinates in three dimensions, the equations of motion of the comet take the same form as (6), but with the addition of a third component parallel to the Z-axis. Using (18), the components of acceleration become

$$\begin{aligned} \frac{d^2x}{dt^2} &= -\mu . \frac{x}{r^3} \\ \frac{d^2y}{dt^2} &= -\mu . \frac{y}{r^3} \\ \frac{d^2z}{dt^2} &= -\mu . \frac{z}{r^3} \qquad \ldots\ldots\ldots\ldots (50) \end{aligned}$$

and these may be taken in pairs to give

$$x \cdot \frac{d^2y}{dt^2} - y \cdot \frac{d^2x}{dt^2} = 0$$

$$y \cdot \frac{d^2z}{dt^2} - z \cdot \frac{d^2y}{dt^2} = 0$$

$$z \cdot \frac{d^2x}{dt^2} - x \cdot \frac{d^2z}{dt^2} = 0 \quad \ldots\ldots\ldots\ldots\ldots(51)$$

which on integration give

$$x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} = h.R_z$$

$$y \cdot \frac{dz}{dt} - z \cdot \frac{dy}{dt} = h.R_x$$

$$z \cdot \frac{dx}{dt} - x \cdot \frac{dz}{dt} = h.R_y \quad \ldots\ldots\ldots\ldots\ldots(52)$$

These equations clearly correspond to (8) and give the projected areal velocities on the three principal planes, the *R*'s being, as before, the direction cosines of the pole of the orbit. The three equations of (52) may be combined to give

$$R_x.x + R_y.y + R_z.z = 0 \quad \ldots\ldots\ldots\ldots\ldots(53)$$

which is the equation of a plane through the origin. Since the motion is in a plane, the position of the comet may be referred to a system of two co-ordinates in this plane, and the resulting equations (6) when integrated in polar co-ordinates, give the equation of the conic (12). In this process, six constants of integration will appear, and these are to be identified with the six orbital elements. Thus in order to compute an orbit from observations, or to determine the future motion of the comet, six independent measurements must be provided.

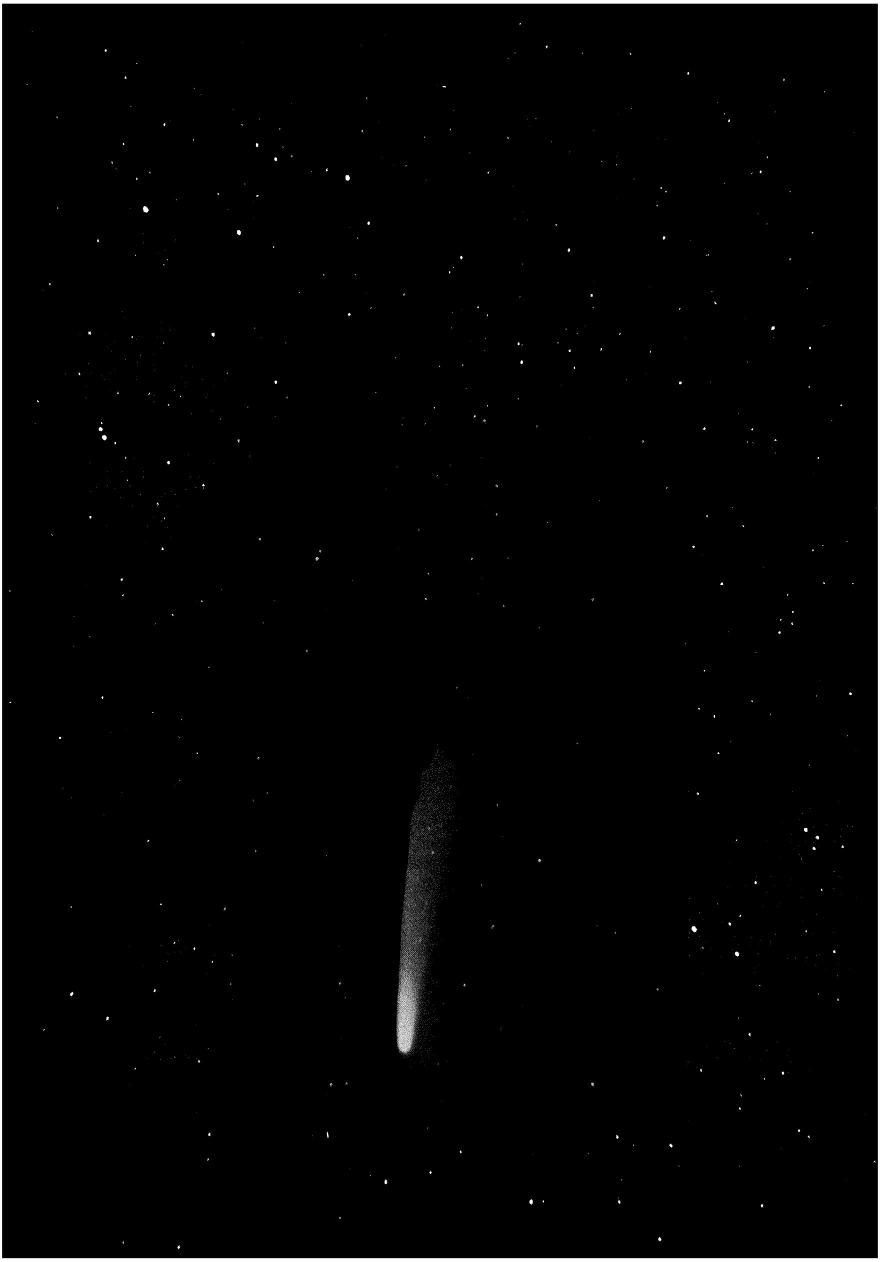
Computing an orbit

In computing an orbit for a newly discovered comet three measures of the position of the comet at given times are sufficient in most cases. The problem is one of singular complexity, and in modern practice is solved either by Leuschner's adaptation of Laplace's method, or by Merton's modification of Gauss's method. (See list of books, p. 33.) Subsequent observations enable differential corrections to be applied to the orbit, and a *definitive orbit* is, in many cases, obtained by making use of every available observation over a lengthy period, and including perturbations. With a sufficiently accurate orbit as a starting point, it is then possible to compute the perturbations which the comet undergoes during the next revolution ; and these perturbations are carried forward to a

suitable date near the next perihelion passage, so as to obtain the orbital elements which will prevail at that time. The ellipse represented by these elements is not the true path of the comet, but is an instantaneous ellipse which is tangent to the variable conic in which the comet is actually travelling, while the velocity at the point of tangency is the same in both curves. The instantaneous ellipse is thus an *osculating orbit* for a given moment only, and this instant of time is the *date* (*or epoch*) *of osculation.*

The ephemeris computed from the osculating orbit is intended as a finding ephemeris only, and as it serves merely as a guide to the observer, no great accuracy is required. The position of a comet is normally obtained by comparison with neighbouring stars whose places are known for the equinox of the beginning of the year. The position of the equinox (♈ in Fig. 5) is affected by precession, which causes it to retrograde along the ecliptic at the rate of 50 seconds per annum, so that all longitudes increase by this amount. For this reason, all the angles used in the calculation of the ephemeris must be corrected for precession to the equinox required. This is usually that of the beginning of the year, but in the computation of perturbations, the position of the comet must be obtained at regular intervals during at least one revolution, and this type of work is always carried out with a fixed equinox, usually that of 1950·0. The work is facilitated by using the co-ordinates of the planets given for this equinox in the volumes of *Planetary Co-ordinates*, which also give convenient tables for the application of the precessional corrections.

The orbit of a periodic comet of only one appearance is always liable to errors. Even a definitive orbit based, perhaps, on a great number of observations, may be incorrect, particularly with respect to the mean motion of the comet. The observations are used to correct a preliminary orbit, the corrections being obtained by a least-squares solution of a number of observational equations. Such a solution has the effect of making the sum of the squares of the residual errors as small as possible—in other words, the solution may be the best orbit mathematically speaking, but it is not necessarily the correct orbit. The only sound orbits, therefore, are those in which the data for one apparition are linked with those for a previous one, for in these circumstances an accurate value of the mean motion may be obtained. The point was clearly illustrated by Merton (1927) in his work on Comet Schaumasse. The value of n obtained from the least square solution was $439''{\cdot}476 \pm 0''{\cdot}7$. The probable error, however, was fictitious, for as so often happens in this type of work, the final equations contained coefficients with only two significant figures, so that the results were unreliable. By linking the 1911 and 1919 apparitions of the comet, Merton showed that a value of $446''{\cdot}811$ for n was more likely. The resulting prediction for the 1927 return made the time of



THE " ECLIPSE " COMET, 1948

A typical bright comet, discovered during the solar eclipse of 1948 November 6. Exposure 5 minutes.

(*Rondanina, Montevideo*)

Facing p. 30

perihelion passage no less than 49 days earlier, and enabled the comet to be recovered although the circumstances were decidedly unfavourable.

The ephemeris

In the calculation of the ephemeris itself, it is convenient to express the co-ordinates of the comet in terms of the eccentric anomaly rather than the true anomaly.

$$\begin{aligned} \text{Since } r\cos v &= a(\cos E - e) \\ \text{and } r\sin v &= b\sin E \end{aligned}$$

these may be combined with (49) to give

$$\begin{aligned} x &= aP_x\,.\,(\cos E - e) + bQ_x\,.\,\sin E \\ y &= aP_y\,.\,(\cos E - e) + bQ_y\,.\,\sin E \\ z &= aP_z\,.\,(\cos E - e) + bQ_z\,.\,\sin E \qquad \ldots\ldots\ldots\ldots(54) \end{aligned}$$

and these are the equations which are in general use for the computation of the ephemeris of a comet of moderate eccentricity. (Gibbs, 1888 ; Adams, 1922. See also Appendix 1.)

The geocentric co-ordinates are found by addition of the solar co-ordinates X, Y, Z, which are given for every day in the year in the *Nautical Almanac* (or *American Ephemeris*), for both of the equinoxes referred to above. Then if ξ, η and ζ are the geocentric equatorial co-ordinates of a comet whose right ascension α and declination δ are required,

$$\begin{aligned} \xi &= x + X = \Delta\cos\delta\cos\alpha \\ \eta &= y + Y = \Delta\cos\delta\sin\alpha \\ \zeta &= z + Z = \Delta\sin\delta \qquad \ldots\ldots\ldots\ldots\ldots(55) \end{aligned}$$

where Δ is the distance of the comet from the earth. The value of r, computed from (36) is also given, since this quantity is of value in photometric work. The ephemeris is given as a rule at ten-day intervals (I.A.U., 1948), the dates being chosen so that they represent the midnights following an integral Julian date that is exactly divisible by 10. An example of a typical ephemeris is given in Table 1, and is taken from the British Astronomical Association's *Handbook* for 1948. This comet was recovered on 1948 May 14 within 3′ of the predicted place by H. M. Jeffers at Lick Observatory. Observations indicated that it was some two magnitudes fainter than had been predicted. The variations in R.A. and Dec. for a delay in perihelion passage of one day are given to enable a continuous correction to be made to the ephemeris after discovery. The variation is generally computed by adding the solar co-ordinates of date to the comet's co-ordinates for the date ten days earlier. The positions computed from this new set of figures give the changes for T ten days later. This variation is, of course, a mere approximation, since it takes no account of the changes in a and n that necessarily result from a change in T.

TABLE 1

COMET FORBES, 1929 II

By F. R. Cripps

This comet has made two previous appearances—in 1929 and 1942. Perturbations by Jupiter and Saturn have been applied to the orbit of the 1942 *Handbook*, T being corrected by $-$ 0·5 day.

T	1948 September 16·194 U.T.	e	0·552735
ω	259°·741 } 1950·0	a	3·45482
☊	25·445 } 1950·0	n	0°·153485
i	4·621 } 1950·0	P	6·42154 years

Equatorial constants for 1948·0 :—

$$x = +\ 0{\cdot}898617\ (\cos E - e) + 2{\cdot}778218 \sin E$$
$$y = -\ 2{\cdot}941168 \qquad\qquad + 0{\cdot}708487$$
$$z = -\ 1{\cdot}574103 \qquad\qquad + 0{\cdot}262229$$

Date 1948	R.A. 1948·0	Dec. 1948·0	r	Δ	Variation for $\Delta T = +1$ day $\Delta\alpha$	$\Delta\delta$	Mag.
	h m	° ′			m	′	
Feb. 3	13 52·1	− 9 44	2·585	2·170	− 1·12	+ 7·9	16·6
11	13 57·8	10 21	2·537	2·021	1·24	8·5	
19	14 02·4	10 53	2·488	1·878	1·36	9·2	
27	14 05·7	11 19	2·439	1·741	1·49	10·0	
Mar. 6	14 07·5	11 39	2·389	1·612	1·62	10·9	15·6
14	14 07·6	− 11 53	2·340	1·493	− 1·76	+ 11·8	
22	14 05·9	12 00	2·291	1·385	1·91	12·8	
30	14 02·3	12 00	2·243	1·291	2·04	13·9	
Apr. 7	13 57·0	11 53	2·194	1·210	2·15	14·9	14·6
15	13 50·2	11 40	2·146	1·144	2·24	15·8	
23	13 42·4	− 11 23	2·098	1·096	− 2·27	+ 16·6	
May 1	13 34·3	11 07	2·051	1·060	2·26	17·1	
9	13 26·7	10 52	2·005	1·041	2·21	17·3	13·9
17	13 20·3	10 42	1·959	1·034	2·12	17·2	
25	13 15·9	10 42	1·915	1·040	2·04	17·0	
June 2	13 13·6	− 10 53	1·872	1·054	− 1·92	+ 16·6	
10	13 13·9	11 16	1·831	1·075	1·84	16·2	13·6
18	13 16·8	11 53	1·791	1·101	1·77	15·8	
26	13 22·1	12 41	1·753	1·130	1·71	15·3	
July 4	13 29·9	13 41	1·718	1·162	1·69	14·9	
12	13 39·9	− 14 51	1·685	1·195	− 1·68	+ 14·5	13·5
20	13 52·0	16 09	1·655	1·228	1·68	14·1	
28	14 06·2	17 33	1·628	1·262	1·71	13·7	
Aug. 5	14 22·3	19 01	1·604	1·296	1·76	13·2	
13	14 40·2	− 20 30	1·584	1·331	− 1·82	+ 12·6	13·4

Magnitudes computed from $m = 10{\cdot}8 + 10 \log r + 5 \log \Delta$

BIBLIOGRAPHY

A. BOOKS

ANDOYER, M. H., 1923, *Cours de Méchanique Céleste* (Paris : Gauthier-Villers).

BAUSCHINGER, J., 1928, *Die Bahnbestimmung der Himmelskörper*, 2nd Edition (Leipzig : Engelmann).

CRAWFORD, R. T., 1930, *Determination of orbits of Comets and Asteroids* (New York : McGraw, Hill).

HERGET, P., 1948, *The Computation of Orbits* (Cincinnati : printed privately).

MOULTON, F. R., 1902, *Celestial Mechanics* (New York : Macmillan).

OPPOLZER, T. R. v., 1882, *Bahnbestimmung der Cometen und Planeten* (Leipzig : Engelmann. Also a French translation by Pacquier, Paris : Gauthier-Villers).

PLUMMER, H. C., 1918, *Dynamical Astronomy* (Cambridge : University Press).

SMART, W. M., 1944, *Spherical Astronomy*, 4th edition (Cambridge : University Press).

STRACKE, G., 1929, *Bahnbestimmung der Planeten und Kometen* (Berlin : Springer).

TISSERAND, F. F., 1889–1896, *Traité de Méchanique Céleste*, 4 vols. (Paris : Gauthier-Villers).

WATSON, J. C., 1900, *Theoretical Astronomy* (Philadelphia : Lippincott).

WILLIAMS, K. P., 1934, *Calculation of the orbits of Asteroids and Comets* (London : Williams and Norgate).

B. TABLES—(Also given in many of the books above.)

ASTRAND, J. J., 1890, *Hülfstafeln zur Auflösung des Kepler'schen Problems* (Leipzig : Engelmann).

BAUSCHINGER, J., 1901, *Tafeln zur Theoretischen Astronomie* (Leipzig : Engelmann).

MØLLER, J. P., 1932, *Table giving tan v/2 in parabolic motion for M = 275 to 4515* (*Pub. Copenhagen Obs.*, No. 82).

SCHLESINGER, F., and UDICK, S., 1912, *Tables for the true anomaly in elliptical orbits* (*Pub. Allegheny Obs.*, II, No. 17, 155).

STRÖMGREN, B., 1929, *Formeln und Tafeln zur Bestimmung parabolischer Bahnen* (*Pub. Copenhagen Obs.*, No. 66).

STRÖMGREN, B., 1927, *Tables giving tan v/2 in parabolic motion , M = 0 to 285, Memoirs B.A.A.*, **27**, Part 2.

C. REFERENCES

ADAMS, C. E., 1922, *Brit. Astr. Assocn. J.*, **32**, 231.

GIBBS, J. W., 1888, *Mem. Nat. Acad. Sci., Washington*, **IV**, pt. 2, 81.

INNES, R. T. A., 1929, *Mon. Not. Roy. Astr. Soc.*, **89**, 422.

MERTON, G., 1925, *Mon. Not. Roy. Astr. Soc.*, **85**, 693.

MERTON, G., 1927, *Mon. Not. Roy. Astr. Soc.*, **87**, 565.

MERTON, G., 1929, *Mon. Not. Roy. Astr. Soc.*, **89**, 451.

MØLLER, J. P., 1933, *Mon. Not. Roy. Astr. Soc.*, **93**, 778 (= *Pub. Copenhagen Obs.*, No. 90).

PORTER, J. G., 1949, *Mon. Not. Roy. Astr. Soc.*, **109**, 409.

SMILEY, C. H., 1930, *Astr. J.*, **40**, No. 4, 1.

WOOD, H., 1950, *J. Roy. Soc. New South Wales*, **83**, 150 and 181.

CHAPTER III

Cometary Statistics

The standard catalogue of cometary orbits is that of Galle (1894) and its continuation by Crommelin (1925, 1932). Galle's list gives all published orbits of comets up to the year 1894, but makes no attempt to discriminate between them. Crommelin's lists are definitely superior in giving only the best available orbits, while the definitive orbits are clearly marked. These lists terminate with the comets of 1931, but data for comets after that date are to be found in the annual reports on comets in the *Vierteljahresschrift der Astronomische Gesellschaft* (up to 1940), and in the *Monthly Notices* of the Royal Astronomical Society. Collecting the best available orbits for all comets from these sources, and rejecting a few which have poorly determined elements, a list is obtained of 737 orbits with some claim to accuracy. It is proposed in this chapter to discuss some of the statistical information which may be derived from this collection.

Distribution of perihelion distances

If in the first place the perihelion distance q is studied, it is found that the values for various comets are not distributed at random, since there is a decided tendency for the majority of observed comets to have values of q in the neighbourhood of unity (see Fig. 11). This is clearly due to the

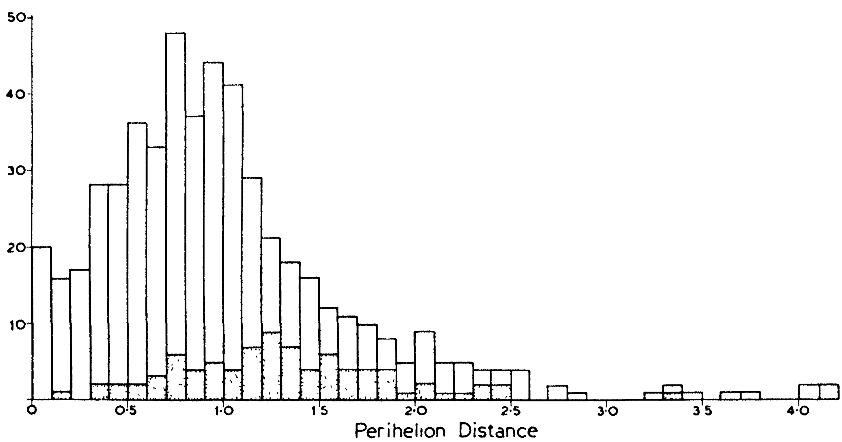


FIG. 11.—Distribution of perihelion distance in cometary orbits. (The shaded areas represent short-period comets.)

fact that we see only those comets that come sufficiently close to the Earth and the Sun to be visible in our instruments. Moreover, the brighter comets are usually those with small perihelion distances, and the distribution shows a bias due to an excess of such small values. The orbits having the smallest and largest known values of q are given in Table 2.

TABLE 2

Greatest and least values of perihelion distance.

Comet	q	Comet	q
1925 I	5·523	1880 I	0·00549
1948 *j*	4·709	1843 I	0·00553
1925 VI	4·179	1680	0·00622
1942 VIII	4·113	1945 *g*	0·00631
1729	4·051	1881 II	0·00775
1936 I	4·043	1887 I	0·00966

There are in all 170 comets with values of q between 0·7 and 1·1 and if this represented a uniform distribution, it would imply that cometary perihelia are distributed at a frequency of 13·5 per unit volume of space. This would give approximately 5 million comets with perihelia inside the orbit of Neptune, and such a value would appear, on any assumptions, to be out of the question. Modern views, however, would regard this number as a small fraction of the total numbers of comets that must exist.

The fact that the list of comets is biased is not the only obstacle to a straightforward statistical analysis of the figures it contains. The comets of the early years were all bright naked-eye comets, and the improvements in instrumental technique have made progressive changes in the character of the results obtained. Again, the elements of a cometary orbit are supposed to be independent, but this is by no means the case in practice. Thus in an elliptical orbit of small eccentricity, the exact position of the perihelion point is ill-defined because the curvature is almost that of a circle. In this case, therefore, T and ω will be subject to large probable errors. Similarly, in an orbit of very small inclination, the position of the node is always uncertain, so that Ω and ω are likely to be in error. A more serious matter arises from the restricted values of q which have been mentioned above. Since $q = a(1 - e)$ it follows that a restricted range of values of q (in the neighbourhood of unity) necessarily entails a correlation between a and e in the majority of orbits. Thus a large value of a is always associated with a large value of e, a fact which is at once obvious in any comet catalogue. The comet list is thus far from having a random distribution (in the statistical sense), but it is the only list available, and if its peculiarities are remembered, it is capable of providing some useful information.

The three types of orbit

It would seem to be a simple matter to divide the orbits into three groups of parabolic, elliptical or hyperbolic orbits, but such a division becomes largely a matter for individual choice. It is only in the case of comets having small values of a that the elliptical nature of the orbit is at all well defined. The great majority of elliptical orbits have values of e so close to unity that the huge elongated ellipses are almost indistinguishable from parabolas. In such cases it is common to find the orbit given in the first instance as a parabola, and only a definitive orbit—one which makes use of every available observation—can serve to differentiate between the three types of orbit. Even a definitive orbit, as will be seen later, may give an entirely false impression, for it is merely an osculating orbit, and cannot define the entire shape of the orbit unless accurate perturbations are computed as well. It is thus a matter of personal choice as to where to draw the line between the elliptical orbit of a comet that must necessarily return to the Sun, and the near-parabola with its vague possibilities for the future.

It has been thought best in the present work to include among the class of *short-period comets* all those comets which have made more than one return to the Sun during the time in which accurate records have been kept. This sets an upper limit of about 200 years for the period of such comets. The list of 737 comets is then found to refer to 525 individual comets as follows :

274 parabolic orbits
52 hyperbolic orbits
199 elliptical orbits.

The last group is of the greatest interest, and is composed of

114 comets with periods greater than 200 years,
43 comets which have made more than one return,
42 short-period comets of only one appearance.

Approximate data for the short period comets are given in Tables 3 and 4 on pages 37 and 38.

Long period comets

The 114 comets which have been assigned elliptical orbits with periods of more than 200 years make an interesting study. Some of these orbits are so large that the periods are given in millions of years. Thus Comet 1910 I is quoted as having a period of 3,910,000 years, but it must be emphasised that no reliance whatever should be placed on such figures as

TABLE 3

Short period comets of more than one apparition.

Comet		N	P	q	e	ω	Ω	i
			yrs.			°	°	°
1950 *e*	Encke	43	3·30	0·338	0·847	185·2	334·7	12·4
1907 III	Tuttle-Giacobini	2	4·13	1·147	0·554	36·0	167·8	13·6
1947 *a*	Grigg-Skjellerup	7	4·90	0·854	0·704	356·4	215·4	17·6
1946 *b*	Tempel (2)	10	5·31	1·393	0·542	190·9	119·4	12·4
1927 I	Neujmin (2)	2	5·43	1·338	0·567	194·7	328·0	10·6
1879 I	Brorsen (1)	5	5·46	0·590	0·810	14·9	101·3	29·4
1908 II	Tempel-Swift	4	5·68	1·153	0·638	113·7	290·3	5·4
1894 IV	de Vico-Swift	3	5·86	1·392	0·572	296·6	48·8	3·0
1879 III	Tempel (1)	3	5·98	1·771	0·463	159·5	78·8	9·8
1945 *a*	Pons-Winnecke	14	6·15	1·159	0·655	170·1	94·5	21·7
1945 *b*	Kopff	6	6·18	1·496	0·556	31·5	253·1	7·2
1948 *e*	Forbes	3	6·42	1·545	0·553	259·7	25·4	4·6
1909 III	Perrine	2	6·45	1·173	0·662	166·9	242·3	15·7
1947 *l*	Schwassmann-Wachmann (2)	4	6·53	2·144	0·384	358·1	126·0	3·7
1946 *c*	Giacobini-Zinner	6	6·59	0·996	0·717	171·8	196·2	30·7
1852 III	Biela	6	6·62	0·861	0·756	223·3	245·9	12·6
1950 *d*	Daniel	4	6·66	1·465	0·586	7·2	69·7	19·7
1950 *a*	d'Arrest	10	6·70	1·378	0·612	174·4	143·6	18·1
1926 V	Finlay	5	6·85	1·059	0·707	320·6	45·3	3·4
1906 III	Holmes	3	6·86	2·122	0·412	14·3	331·7	20·8
1932 IV	Borrelly	5	6·87	1·385	0·617	352·5	77·1	30·5
1946 *e*	Brooks (2)	8	6·96	1·879	0·484	195·6	177·7	5·5
1947 *g*	Whipple	3	7·41	2·449	0·356	190·1	188·6	10·2
1947 *f*	Faye	13	7·44	1·660	0·564	200·3	206·4	10·6
1949 *f*	Reinmuth (1)	3	7·69	2·037	0·477	12·9	123·6	8·4
1942 VII	Oterma (3)	—	7·89	3·390	0·144	354·8	155·2	4·0
1943 V	Schaumasse	4	8·16	1·193	0·705	51·0	86·7	12·0
1950 *c*	Wolf (1)	9	8·42	2·498	0·396	161·1	203.9	27·3
1944 II	Comas Solá	3	8·50	1·766	0·576	38·9	65·7	13·7
1949 *h*	Väisälä (1)	2	10·52	1·752	0·635	44·3	135·5	11·3
1938 I	Gale	2	10·99	1·183	0·761	209·1	67·3	11·7
1939 X	Tuttle	8	13·61	1·022	0·821	207·0	269·8	54·7
1941 VI	Schwassmann-Wachmann (1)	2	16·15	5·523	0·136	356·2	322·0	9·5
1948 *f*	Neujmin (1)	3	17·93	1·547	0·774	346·7	347·1	15·0
1928 III	Crommelin	4	27·91	0·745	0·919	195·9	250·1	28·9
1942 IX	Stephan-Oterma	2	38·96	1·596	0·861	358·3	78·5	17·9
1913 VI	Westphal	2	61·73	1·254	0·920	57·1	346·8	40·9
1919 III	Brorsen-Metcalf	2	69·06	0·485	0·971	129·5	310·8	19·2
1884 I	Pons-Brooks	2	71·56	0·776	0·955	199·2	254·1	74·0
1887 V	Olbers	2	72·65	1·199	0·931	65·3	84·5	44·6
1910 II	Halley	29	76·03	0·587	0·967	111·7	57·3	162·2
1939 VI	Herschel-Rigollet	2	156·0	0·748	0·974	29·3	355·1	64·2
1907 II	Grigg-Mellish	2	164·3	0·923	0·969	328·4	189·2	109·8

This list gives approximate elements of the orbits of comets at their last apparition, and is complete up to the end of 1950. Comet P/Oterma (3) is included since it is visible at opposition each year.

N = Number of appearances.

TABLE 4

Short period comets of only one apparition.

Comet		P	q	e	ω	☊	i
		yrs.			°	°	°
1949 *g*	Wilson-Harrington	2·3	1·028	0·412	91·9	278·6	2·2
1766 II	Helfenzrieder	3·89	0·411	0·834	180·4	71·6	7·8
1945 *c*	du Toit	4·56	1·235	0·551	203·4	358·7	6·5
1948 *n*	Honda-Mrkos-Pajdušáková	5·00	0·558	0·809	183·8	233·0	13·1
1819 IV	Blanpain	5·10	0·892	0·699	350·1	77·4	9·1
1884 II	Barnard (1)	5·40	1·280	0·584	301·0	5·1	5·5
1930 VI	Schwassmann-Wachmann (3)	5·43	1·011	0·672	192·3	76·8	17·4
1743 I	Grischow	5·44	0·862	0·721	6·4	86·9	1·9
1941 VII	du Toit-Neujmin-Delporte	5·52	1·305	0·582	69·3	229·6	3·3
1886 IV	Brooks (1)	5·60	1·327	0·579	176·8	53·5	12·7
1770 I	Lexell	5·60	0·674	0·786	224·3	132·0	1·6
1939 VIII	Kulin	5·64	1·749	0·448	292·8	137·6	4·8
1783	Pigott	5·89	1·459	0·552	354·6	55·7	45·1
1916 I	Taylor	6·37	1·558	0·546	354·8	113·9	15·5
1890 VII	Spitaler	6·37	1·817	0·471	13·3	45·1	12·8
1947 *j*	Reinmuth (2)	6·57	1·863	0·469	43·3	297·4	7·1
1892 V	Barnard (2)	6·63	1·434	0·594	169·9	206·4	31·3
1896 V	Giacobini (1)	6·65	1·454	0·588	140·5	193·4	11·4
1918 III	Schorr	6·71	1·882	0·471	278·6	118·0	5·6
1949 *d*	Johnson	6·85	2·248	0·377	206·1	118·2	13·9
1895 II	L. Swift	7·22	1·298	0·652	167·8	170·3	3·0
1948 *b*	Wirtanen	7·25	1·648	0·560	344·0	86·3	13·5
1894 I	Denning (2)	7·42	1·147	0·698	46·2	84·4	5·5
1948 *i*	Ashbrook-Jackson	7·47	2·311	0·395	348·9	2·4	12·5
1924 IV	Wolf (2)	7·49	2·431	0·365	177·8	260·3	23·8
1949 *e*	Shajn-Schaldach	7·76	2·302	0·413	217·2	168·0	6·7
1906 VI	Metcalf	7·77	1·632	0·584	200·0	194·6	14·6
1881 V	Denning (1)	8·49	0·725	0·828	312·5	65·9	6·9
1936 IV	Jackson-Neujmin	8·53	1·462	0·650	197·3	164·2	13·3
1889 VI	Swift	8·92	1·356	0·685	69·8	330·4	10·3
1929 III	Neujmin (3)	10·90	2·040	0·585	140·5	158·2	3·7
1846 VI	Peters	13·38	1·529	0·729	339·6	260·4	30·7
1944 III	du Toit	14·87	1·277	0·789	257·0	22·4	18·8
1866 I	Tempel	33·16	0·977	0·905	171·0	231·4	162·7
1827 II	Pons-Gambart	46·0 ?	0·806	0·940	19·2	317·5	136·5
1883 II	Ross	64·6	0·309	0.981	137·6	264·3	114·7
1921 I	Dubiago	67·0	1·116	0·932	97·4	66·1	22·3
1846 IV	de Vico	75·7	0·664	0·963	12·9	77·6	85·1
1942 II	Väisälä	85·5	1·244	0·934	335·2	171·6	38·0
1862 III	Swift-Tuttle	119·6	0·963	0·960	152·8	137·5	113·6
1889 III	Barnard	128·3	1·102	0·957	60·1	271·0	31·2
1917 I	Mellish	145·0	0·190	0·993	121·3	87·5	32·7

this. In the first place, such a figure, even if its reality were beyond doubt, is of no interest to the astonomer. It is the value of a that matters, and this, as has been shown, is correlated with e. Such a large value for the period simply means that e is so close to unity that the orbit is nearly parabolic—

so nearly parabolic, in fact, that the slightest change in the orbit caused by the perturbations by the major planets, may alter it to a smaller ellipse, or change it to a hyperbola. This has been shown clearly in the work of Strömgren (1914) who, by following backwards in time the perturbations of certain comets with well-defined orbits, eventually reached figures which showed no further change due to perturbations. If the velocity and radius-vector of the comet are known, then its semi-major axis a is given by $V^2 = \frac{2}{r} - \frac{1}{a}$ in suitable units. Thus the changes in $1/a$ in such cases as this, where a is large, are a sensitive guide to the changes produced by perturbations ; for $1/a$ becomes zero for a parabola, is positive for an ellipse, and is negative for a hyperbola.

The major part of Strömgren's work was concerned with the values of $1/a$ in hyperbolic orbits, and will be referred to again, but in the course of his work, he makes the important point that the published values for the period in nearly-parabolic orbits are quite worthless and misleading. All comets are constantly under the perturbative influence of the planets, and the published elements are merely osculating orbits of value to the mathematical astronomer. The point may be illustrated by taking an example from Strömgren's early work. Comet 1882 II had a definitive orbit with a value of $e = 0{\cdot}999\ 9078 \pm 0{\cdot}000\ 0021$ and of $\log q = 7{\cdot}889\ 3177 \pm 0{\cdot}000\ 0511$, and these values lead to a value of $1/a = +\ 0{\cdot}011\ 8963 \pm 0{\cdot}000\ 2710$. The effect of perturbations on this comet were studied by Strömgren by computing the attractions of Jupiter and Saturn on the comet backwards in time to 1868, at which period the comet was so far removed from these planets that their attractions could be ignored. The comet was then found to have $1/a = 0{\cdot}012149$, the effect of the change being to alter the period as given in the catalogues (771 years) to an original figure of 747 years.

Families of comets

It was early noticed that there is a tendency among the short-period comets for their aphelia to lie close to the orbits of the major planets. Ignoring the inclination of the orbits, there is a large group of comets associated in this way with the orbit of Jupiter, while smaller groups are similarly connected with Saturn, Uranus and Neptune. The distribution of aphelion-distance is shown in Fig. 12, and this grouping gave rise to the " capture theory " of the origin of short-period comets. The difficulty of otherwise accounting for the remarkable nature of these orbits was met by the theory that comets, originally travelling in much larger orbits, had been constrained by the attraction of a major planet to follow an orbit lying more closely to the ecliptic, the point at which the perturbations

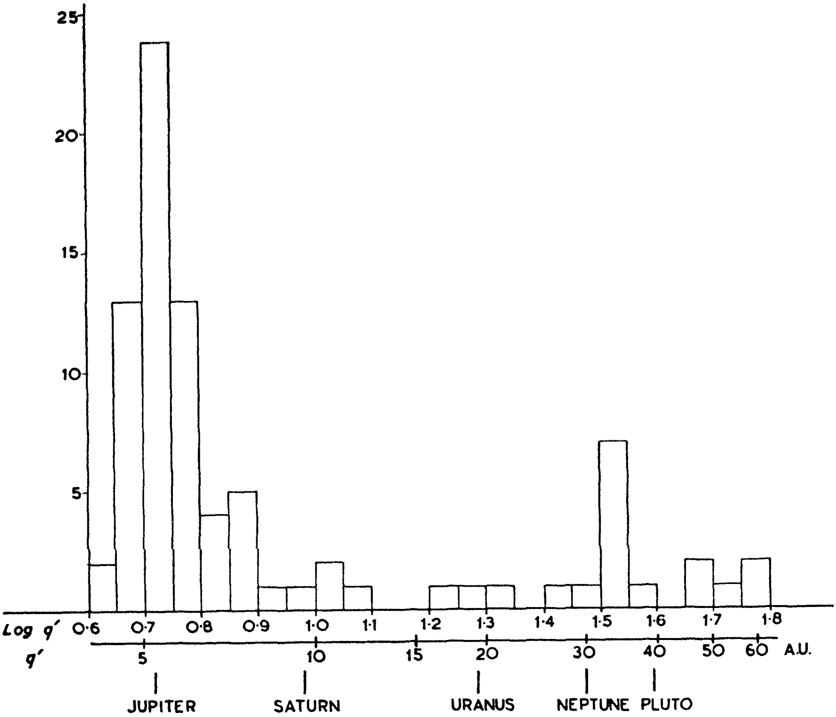


FIG. 12.—Distribution of aphelion distance in short-period orbits.

had taken effect becoming the aphelion. The theory, although attractive, is beset with difficulties. The inherent improbability that such chance encounters should take place in sufficient numbers to create and maintain the growing number of short-period comets was pointed out by Proctor (1884), and other writers have dealt with this matter from the point of view of the mathematical theory of probability. Russell (1920), however, has shown that if the theory has any basis at all, then Jupiter must be mainly responsible. As will be seen from Fig. 13, which illustrates the circumstances of the approach of Halley's comet to the orbit of Neptune, there is actually a distance of 8 units between the two orbits at the point of closest approach, whereas the comet may approach Jupiter's orbit within a distance of one unit. Halley's comet has the least inclination of any of the members of the " Neptune family " of comets, and the effect of Neptune on the others will be even smaller.

Distribution of eccentricities

The eccentricities of cometary orbits are correlated, as has been shown, with the size of the orbit. The long-period comets all have an eccentricity greater than 0·96 :

e	0·96–0·97	0·97–0·98	0·98–0·99	0·99–1·00
No.	2	9	15	88

The short-period comets show a much wider range of distribution of *e*. There are two comets with an eccentricity less than 0·15, and the others range as follows :

e	0·3–0·4	0·4–0·5	0·5–0·6	0·6–0·7	0·7–0·8	0·8–0·9	0·9–1·0
No.	5	9	19	15	11	7	17

The two comets with *e* less than 0·15 are quite exceptional. At the beginning of this century it might well have been said that a comet could be distinguished from a minor planet by the greater eccentricity of its orbit, but this view is no longer tenable. The orbits of these two comets—Schwassmann-Wachmann (1), 1925 II and Oterma, 1942 VII—are shown in Fig. 14.

Distribution of ☊ and *i*

The distribution of the longitudes of the ascending nodes of cometary orbits (Fig. 15) shows at first glance a comparatively uniform scattering of the nodal points around the ecliptic. There is an apparent tendency for

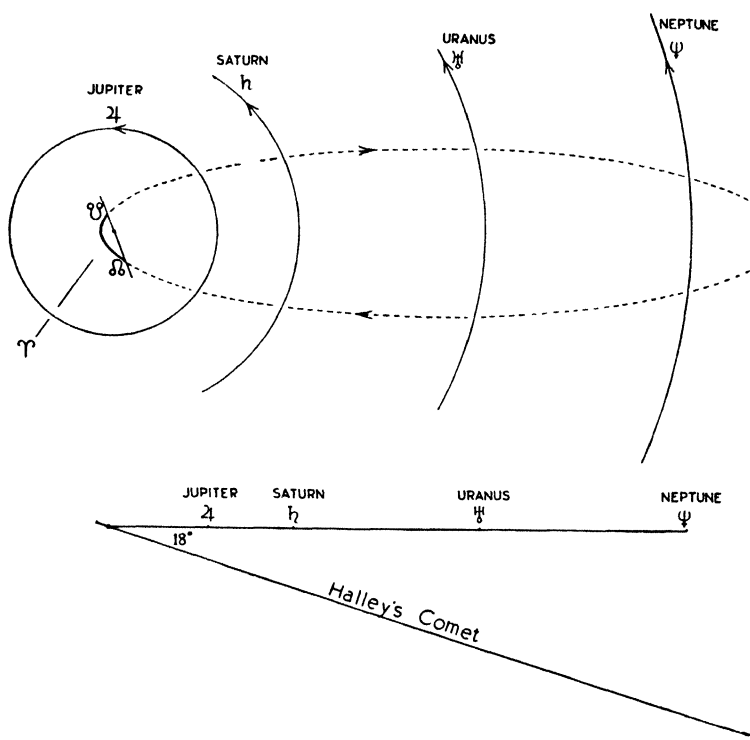


FIG. 13.—The orbit of Halley's comet.

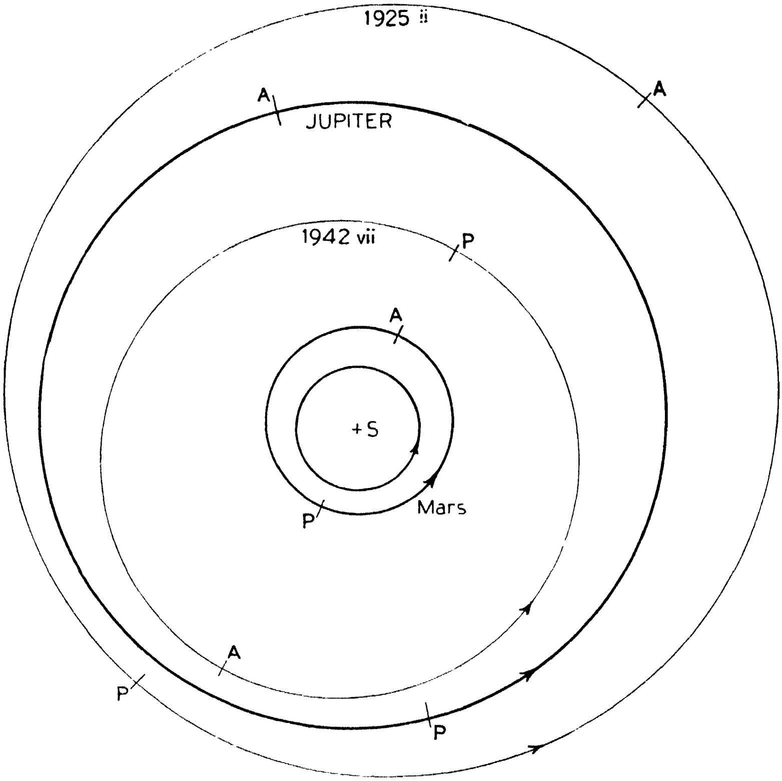


FIG. 14.—The orbits of P/Schwassmann (1), 1925 II and P/Oterma, 1942 VII.

Comet Schwassmann-Wachmann (1) has the smallest eccentricity of any known comet. Its orbit lies entirely between those of Jupiter and Saturn, and is inclined less than 10° to the ecliptic. The comet is remarkable for its sudden and unexplained outbursts of luminosity, sometimes increasing its brightness by more than a hundredfold in a few days. It is observable at each opposition, and has been seen each year since its discovery.

Comet Oterma has a smaller orbit, which lies between those of Mars and Jupiter, and has the small value of 0·144 for its eccentricity. It is observable throughout its orbit, but no unexpected changes in brightness have been noticed. The orbit of this comet, with its small inclination of less than 4°, is strikingly similar to those of the Hilda group of minor planets.

the nodes to concentrate in the regions 72°–96°, 140° and 210°, but otherwise nothing abnormal is evident. If, however, we consider the inclination of the orbit at the same time, we can visualise more clearly the orientation of the orbit planes, and the apparent distribution is no longer uniform. This is, indeed, true of the distribution of inclinations. It is sometimes stated that the inclinations of cometary orbits have all possible values from 0° to 180°, and while this is true, the distribution is by no means a random one, as may seen from Fig. 16. In the case of the short-period comets, there is a marked tendency for the orbits to have small inclina-

tions, so that they lie near the plane of the ecliptic. In addition, the majority have direct motion, and of those comets which have made more than one return to the Sun, only two (Halley's and Grigg-Mellish) have a retrograde motion.

The theoretical random distribution may be visualised by considering the poles of the orbits. If these were distributed at random over the celestial sphere, there would be few near the celestial poles, the number increasing as the ecliptic is approached. The distribution is, in fact, proportional to the areas of zones of the sphere of equal width, i.e., proportional to sin i. The theoretical distribution for the number of comets shown in Fig. 16 is indicated by the dotted sine curve, and it is then seen that the distribution is by no means a random one, there being

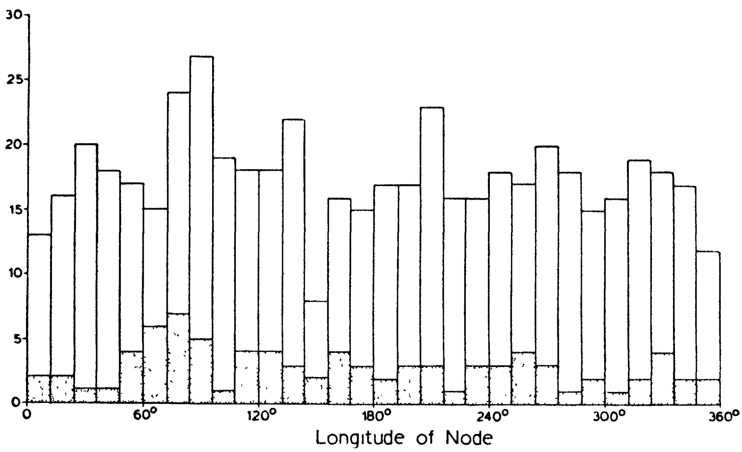


FIG. 15.—Distribution of nodes in cometary orbits. (The shaded areas represent short-period comets.)

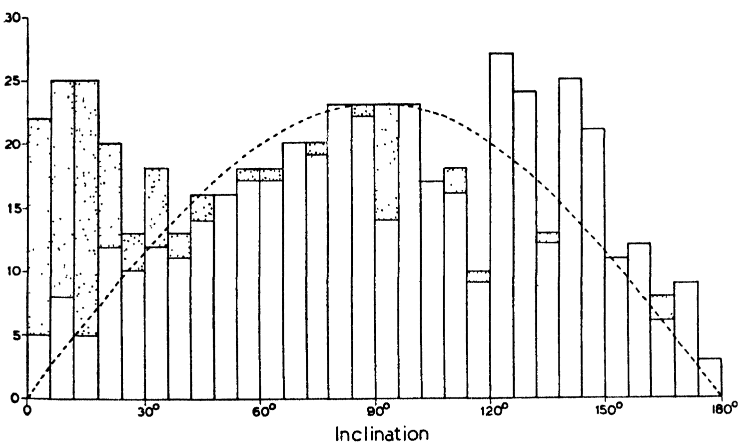


FIG. 16.—Distribution of inclinations in cometary orbits. (The shaded areas represent short-period comets.)

a definite tendency for orbits to crowd towards the ecliptic. This tendency is much greater in the case of short period comets, and is one of the main distinguishing features of this group.

The distribution of the poles themselves is far from being uniform, as may be seen from Tables 5 and 6. The pole of an orbit will lie in longitude ☊–90° and latitude 90°–i, and the numbers in Table 5 show the distribution of long-period and parabolic orbits in 64 equal areas of the celestial

TABLE 5

Distribution of poles of orbits, parabolic and long period only.

Latitude \ Longitude	0°–45°	45°–90°	90°–135°	135°–180°	180°–225°	225°–270°	270°–315°	315°–0°	Totals
+ 90° – 50°	6	3	12	9	5	7	10	5	57
50° – 30°	11	7	7	5	5	6	8	4	53
30° – + 15°	10	2	5	7	9	3	7	3	46
+ 15° – 0°	9	3	9	4	12	4	3	11	55
0° – − 15°	2	6	6	10	8	7	4	3	46
− 15° – 30°	2	3	5	5	4	3	7	4	33
30° – 50°	10	13	5	8	7	14	7	7	71
50° – − 90°	11	8	9	6	7	10	11	17	79
Totals :	61	45	58	54	57	54	57	54	440

TABLE 6

Distribution of poles of orbits, short-period comets only.

Latitude \ Longitude	0°–45°	45°–90°	90°–135°	135°–180°	180°–225°	225°–270°	270°–315°	315°–0°	Totals
+ 90° – 50°	9	11	8	8	5	7	5	19	72
50° – 30°				1		1		2	4
30° – + 15°				1		1			2
+ 15° – 0°								1	1
0° – − 15°									0
− 15° – 30°		1	1	1					3
30° – 50°						1			1
50° – − 90°				1				1	2
Totals :	9	12	9	12	5	10	5	23	85

sphere. There is little to be noticed in this table beyond a tendency for an excess of orbits with retrograde motion (i.e., poles in the southern hemisphere). The short period comets which are tabulated in Table 6 show a very different feature in their obvious excess of small inclinations so that their poles are mainly clustered about the north celestial pole. It is this very marked characteristic which has led to the capture theory of the origin of this class of orbit, and the theory may be extended further to those orbits which have a considerable inclination to the plane of the ecliptic. As Strömgren (1947) has pointed out, the perihelia of such orbits are, in the majority of cases, still close to the ecliptic, because the angle ω lies near 0° or 180°. The actual distance of the perihelion (or aphelion) of the orbit above the plane of the ecliptic is given by $q \sin \omega \sin i$ and even a casual inspection of the data of Tables 3 and 4 will show that the majority of comets have values of i or ω (or both) which lie close to 0° or 180°. This fact is shown more clearly in Table 7, which gives the distribution of short period comets (periods less than 40 years) in ω and i.

Since the aphelia of these comets lie near the orbit of Jupiter, there is every reason for considering that this planet is responsible for the formation of these orbits, although the influence of Jupiter is less well marked here

TABLE 7

Distribution of short-period comets (periods less than 40 years) in ω and i.

Inclination i \ Argument ω	0° / 180° / 180° / 0°			30° / 150° / 210° / 330°			60° / 120° / 240° / 300°			90° / 90° / 270° / 270° Totals
60°–54°			1							1
54°–48°										0
48°–42°	1									1
42°–36°										0
36°–30°	1	1	1							3
30°–24°	2	2								4
24°–18°	2	1	1					1		5
18°–12°	8	8	1	1	1					19
12°–6°	4	6	4	2	3	1	1			21
6°–0°	3	1	1	2	2	1	2	2	2	16
Totals:	21	19	9	5	6	2	3	3	2	70

than it is in the case of the numerous minor planets. These little bodies have orbits in which the distribution of perihelia shows a marked concentration in the region of longitude 13°, which is that of Jupiter's perihelion. As may be seen from Fig. 17, there is a decided tendency for some such effect in the orbits of the short-period comets. The effect is less noticeable owing to the relative paucity of data on comets, or perhaps because the comets are of later formation and move in more eccentric orbits. Strömgren (1947) has pointed out that the effect of Jupiter is on the aphelia of the comets, but there is no reason why a much larger class of comets should not exist, such that their perihelia fall within the influence of Jupiter. He suggests that P/Schwassmann-Wachmann (1) may be an isolated specimen of this class, the other members of which are normally far outside the range of our instruments.

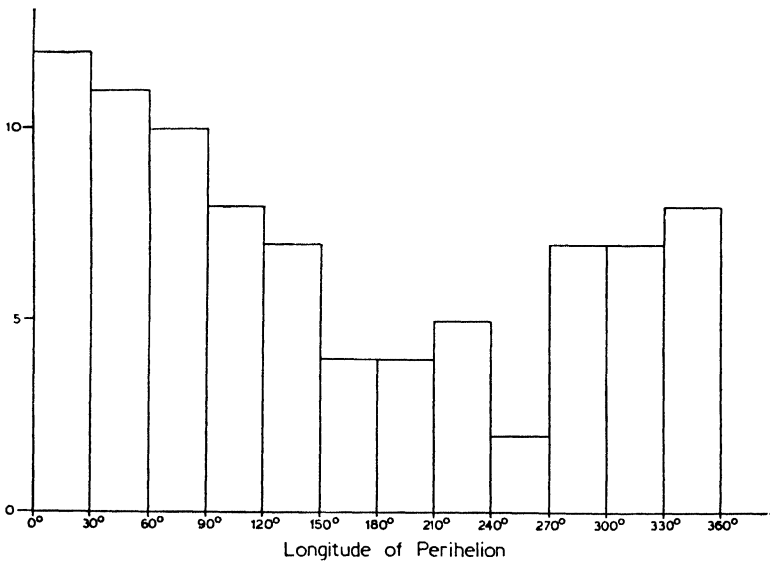


FIG. 17.—Distribution of longitudes of perihelion in short-period comets.

Distribution of aphelia

In considering the long-period comets and those with parabolic orbits it is of more interest to consider the distribution of their aphelia, as this gives some indication of the direction in space from which these bodies have come. The distribution is shown in Table 8 in which the numbers are arranged in 144 roughly equal areas of the celestial sphere. The distribution is seen to be far from uniform, there being a marked concentration of the aphelia in the southern hemisphere and in the region of longitudes 75° to 105°. There is no correlation here between this concentration and the position of Jupiter's aphelion (longitude 193°·5, latitude + 1°·3) nor

TABLE 8

Distribution of Aphelia, long-period and parabolic orbits

Latitude \ Longitude	345°–15°	15°–45°	45°–75°	75°–105°	105°–135°	135°–165°	165°–195°	195°–225°	225°–255°	255°–285°	285°–315°	315°–345°	Totals
+90° – 56°			2	4	3		4	5	4		1	1	24
56° – 42°	3	1	2	5	3	1	1	1		4			21
42° – 30°		4	1	4	1	1	1		1	2	3	1	19
30° – 20°	5	5	4	2		1	2	4	1	4	3	7	38
20° – +10°	4	3	1	3	3			1	3	3	6	4	31
+10° – 0°	2	6	3	7	2	1	3	4		5	1	4	38
0° – −10°	1	2	4	10	10	3	3	3	4	3	6	2	51
−10° – 20°		1	4	7	3	5	3	2	1	7	4	4	41
20° – 30°	1	1	1	3	6	2	3	5	5	3	3	2	35
30° – 42°	1	4	9	10	6	5	5	3	3	4		2	52
42° – 56°	2	4	5	9			8	3	3	7	4	3	48
56° – −90°	4	3	5	2	1	4	1		6	7	7	2	42
Totals :	23	34	41	66	38	23	34	31	31	49	38	32	440

is there any apparent effect due to the motion of the Sun through space. The position of the solar apex is at λ 271°, β + 54°, but if the aphelia are grouped in a similar arrangement to Table 8, but with the solar apex as pole, there are found to be 189 orbits in the hemisphere containing the apex, as against 251 in the opposite half of the celestial sphere. This fact is used as the basis of the argument on page 50 against the extra-solar origin of hyperbolic orbits.

The distribution of aphelia over the celestial sphere was examined in a detailed research by Pickering (1911) who was seeking to establish the presence of unknown planets by the effects which such bodies might produce on the orbits of comets. Although his conclusions can hardly be supported to-day, it must be accepted that these aphelia are not distributed entirely at random, and various possible groupings of comets can be derived from the data.

Comet groups

In his paper, Pickering gives no less than 66 such groups of comets with strikingly similar orbits. The most remarkable example of such a group is

one that caused a good deal of speculation in the last century. It consists of the comets 1843 I, 1880 I, 1822 II and 1887 I, and it is quite likely that the comet of 1668 is also a member of the group. All of these were bright comets, with highly eccentric orbits, and extremely small perihelion distances, and it is quite impossible that any two of them should be appearances of the same comet. It can only be concluded that the comets were all travelling along what was, to all intents and purposes, the same orbit. It must be remembered in this connection that it is impossible for two bodies to continue indefinitely in the same orbit if their masses are different ; and it follows that such a group of comets will always tend to separate eventually, both from one another and from the original orbit, even if perturbations are excluded. It is naturally tempting to speculate on the past history of such a group, which may be regarded as the constituent parts of some parent comet, separated, in the course of many thousands of years, by normal Keplerian forces, as well as by perturbations. The phenomenon of the existence of comet groups is one of great importance, and deserves further study, particularly as the individual particles of a meteor stream behave in exactly the same way. In this case multitudes of minute particles are traversing the same orbit, and there is less opportunity of studying the changes that take place ; but otherwise the analogy is complete.

In Table 9 will be found a list of the more clearly defined comet groups, selected on the basis of similarity in orbital elements and proximity of aphelia.

TABLE 9

Comet Groups.

Comet	π	Ω	i	q	e	Aphelion λ	β
1668	248°.8	358°.6	144°.3	0·0666	1	65°	−33°
1843 I	278·7	1·3	144·3	0·0055	0·99991	100	−35
1880 I	279·9	6·1	144·7	0·0055	1	101	−35
1882 II	276·4	346·0	142·0	0·0077	0·99991	101	−35
1887 I	266·2	324·6	128·5	0·0097	1	99	−42
1945 *g*	270·7	321·6	137·0	0·0063	1	100	−32
1881 IV	334·9	97·0	140·2	0·634	1	148	−33
1898 X	332·8	96·3	140·3	0·756	0·99974	146	−32
1802	332·1	310·2	57·0	1·094	1	143	−18
1900 II	340·4	328·0	62·5	1·015	1·0003	154	−11
1097	332·5	207·5	73·5	0·738	1	185	−52
1774	317·4	180·7	83·3	1·433	1	174	−43
1840 III	324·0	186·0	79·9	0·748	1	177	−41

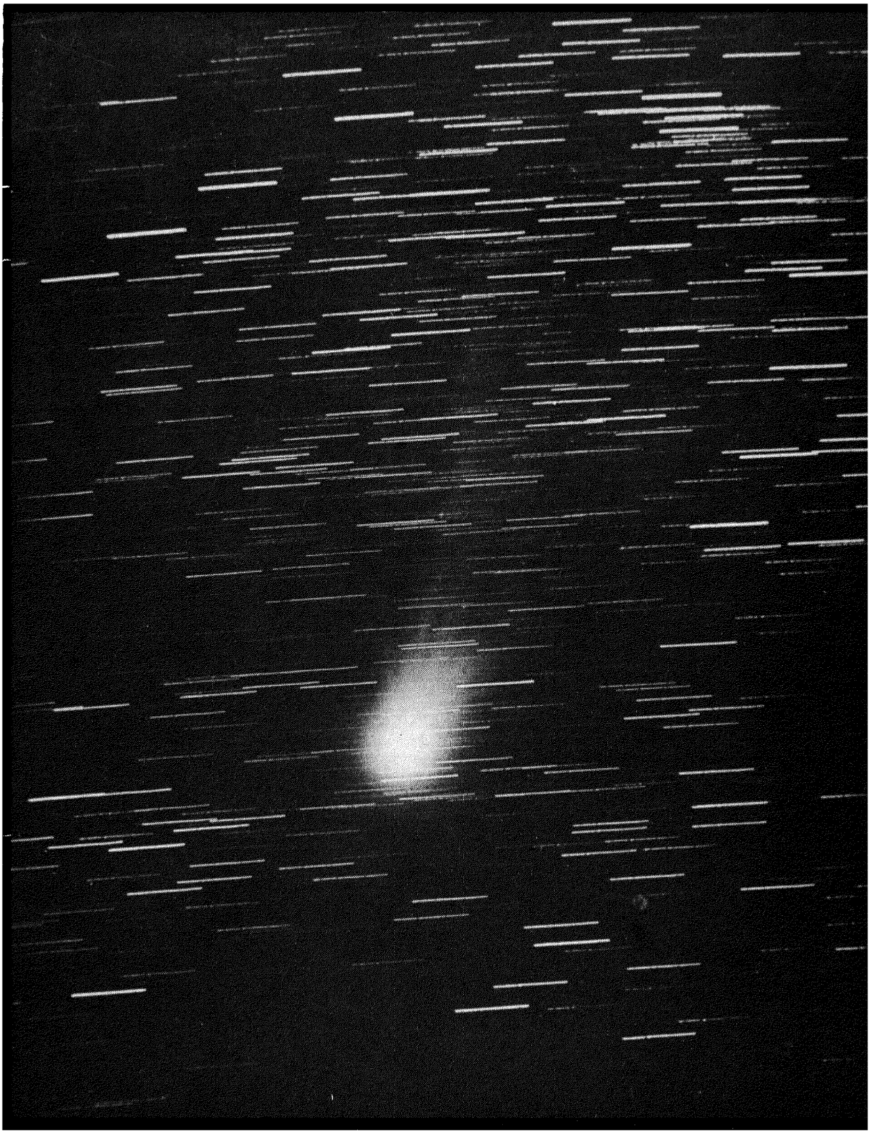
Comet	π	☊	i	q	e	Aphelion λ	β
770	2°·1	88°·9	120°·5	0·603	1	185°	− 59°
1337	2·3	93·0	139·5	0·828	1	182	− 41
1468	1·4	71·1	142·0	0·830	1	186	− 35
1787	7·8	106·9	131·7	0·349	1	183	− 48
1799 I	3·7	99·5	129·1	0·840	1	180	− 51
1652	28·3	88·2	79·5	0·847	1	251	+ 58
1895 III	21·9	83·1	76·2	0·843	1	240	+ 58
1810	63·7	308·8	62·9	0·970	1	264	− 54
1863 V	60·4	304·7	64·5	0·772	1	263	− 54
1845 I	91·3	336·7	46·9	0·905	1·00025	280	− 42
1854 IV	94·4	324·5	40·9	0·799	0·99245	282	− 30
1925 VII	81·0	334·6	49·3	1·566	1·0004	269	− 47
1580	108·4	19·1	64·6	0·602	1	287	− 65
1846 VIII	98·7	4·7	49·7	0·831	1	281	− 50
1890 III	100·0	14·3	63·3	0·764	1	275	− 63
1762	104·1	348·6	85·6	1·009	1	340	− 64
1877 III	102·9	346·1	77·2	1·009	1	322	− 60
1892 VI	157·2	264·5	24·8	0·976	1	336	+ 24
1923 I	166·4	262·0	23·4	0·924	0·9937	346	+ 23
1699	212·2	321·7	109·4	0·749	1	5	− 63
1854 II	213·9	315·5	97·5	0·277	1	348	− 76
1858 IV	226·1	325·0	100·0	0·544	1	13	− 77
1822 I	192·7	177·4	126·4	0·504	1	7	+ 12
1864 I	188·9	175·0	135·0	0·626	1	5	+ 10
1893 IV	187·4	174·9	129·8	0·812	1	3	+ 10
1743 II	247·1	6·1	134·4	0·523	1	58	− 39
1808 II	252·7	24·2	140·7	0·608	1	65	− 28
1857 III	249·6	23·7	121·0	0·367	1	52	− 38
1857 V	250·2	15·0	123·9	0·563	0·9969	54	− 43
1911 VI	273·2	35·2	108·1	0·787	0·9982	62	− 54
1914 I	276·3	32·7	113·0	0·543	1	71	− 55
1748 II	278·7	33·1	67·1	0·625	1	74	+ 57
1849 III	267·1	30·5	66·9	0·894	0·9978	61	+ 50
1902 I	280·7	52·3	66·5	0·451	1	76	+ 44
1790 III	273·7	33·2	116·1	0·798	1	71	− 54
1825 I	273·9	20·1	123·3	0·889	0·9962	82	− 53
1807	270·9	266·8	63·2	0·646	0·9955	89	− 4
1880 V	261·1	249·4	60·7	0·660	1	75	− 10
1881 III	265·3	271·0	63·4	0·769	0·9957	89	+ 5
1433	267·0	96·3	104·0	0·493	1	94	+ 9
1822 IV	271·6	92·7	127·3	1·145	0·9963	92	+ 1
1897 I	274·2	86·5	146·1	1·063	1·0009	93	− 4
1908 III	291·6	103·2	140·2	0·945	1·0007	110	− 5

Hyperbolic orbits

It has been stated above that no less than 52 of the comets in this list have been given hyperbolic orbits. The very existence of a hyperbolic orbit would suggest an extra-solar origin, since the velocity in such an orbit is greater than that which would be acquired in falling freely from infinity. A hyperbolic orbit could only be acquired in two ways—either the comet approaches the Sun from outer space with a velocity of its own, or the comet has increased its velocity (originally nearly parabolic) by the perturbations of the planets. The second of these views is now accepted, although it cannot be said that the first view was ever taken at all seriously. For no orbits are known in which the hyperbolic character is very strongly marked, the eccentricity in all cases being only slightly greater than unity. The greatest values are those of 1886 III (in which $e = 1{\cdot}0130$) and 1926 VII ($e = 1{\cdot}00866$), but neither of these is a definitive orbit. The definitive orbit of 1914 III has a value of $e = 1{\cdot}00367$, but if comets really came to us from outer space, we should expect a much wider range of eccentricities than this.

Again, as Crommelin has pointed out (1929), comets show no preferential direction of approach to the Sun. If they came from outer space, their motion would be compounded with that of the Sun, and there would be an apparent tendency for their directions of approach to crowd towards the hemisphere centred round the solar apex. (See page 47.)

The most convincing argument against the reality of these hyperbolic orbits comes from the work of Strömgren and others on the effects of perturbations in such cases. In his original paper, Strömgren (1914) used those orbits whose hyperbolic nature he considered to be based on sufficiently accurate data, and computed the perturbations backwards in time in the manner already described. It was thus found that the comets were originally travelling in elliptical orbits, which had been altered by planetary perturbations until the motion of the comet became hyperbolic. The work has since been extended by other computers and the results obtained to date are given in Table 10, which is based on that given by Sinding (1948), with some corrections and additions. Comets Carrasco 1931 V and Whipple-Bernasconi-Kulin, 1942a have also been investigated, although not in such complete detail, while some early work by Fayet (1911) has also led to similar conclusions. It will be seen from the table that the change in $1/a$ due to perturbations is of the order of $-\ 0{\cdot}0005$; and as Sinding remarks, these perturbations are of such a nature that Jupiter and the Sun act as one attracting force while the comet is at a great distance, but become separate centres of attraction as the comet nears perihelion.



COMET PERRINE, 1902 III
The photograph shows clearly the transparent coma and tail.

(*Royal Observatory, Greenwich*)

Facing p. 50

TABLE 10

Hyperbolic Orbits.

Comet	Osculating $1/a$	Original $1/a$	$\Delta(1/a)$
1853 III	− 0·0008193	+ 0·0000829	− 0·0009022
1863 VI	− 0·0004949	+ 0·0000166	− 0·0005115
1882 II	+ 0·0118963	+ 0·0121488	− 0·0002525
1886 I	− 0·0006944	− 0·0000071	− 0·0006873
1886 II	− 0·0004770	+ 0·0003166	− 0·0007936
1886 IX	− 0·0005765	+ 0·0000630	− 0·0006395
1889 I	− 0·0006915	+ 0·0000422	− 0·0007337
1890 II	− 0·0002151	+ 0·0000718	− 0·0002869
1897 I	− 0·0008722	+ 0·0000396	− 0·0009118
1898 VII	− 0·0006074	− 0·0000157	− 0·0005917
1902 III	+ 0·0000810	+ 0·0000054	+ 0·0000756
1904 I	− 0·0005040	+ 0·0002165	− 0·0007205
1905 VI	− 0·0001424	+ 0·0006210	− 0·0007634
1907 I	− 0·0004991	+ 0·0000252	− 0·0005243
1908 III	− 0·000732	+ 0·0001581	− 0·0008901
1910 I	+ 0·0002143	+ 0·0006291	− 0·0004778
1914 V	0·0001465	+ 0·0000119	− 0·0001584
1922 II	0·0003806	+ 0·0000038	− 0·0003844
1925 I	− 0·0005665	+ 0·0000540	− 0.0006205
1925 VII	− 0·0002730	+ 0·0001150	− 0·0003880
1932 VI	− 0·0005948	+ 0·0000441	− 0·0006389
1936 I	− 0·000487	+ 0·000205	− 0·000692

Thus we see that not one case of certain hyperbolic motion remains. Of the 22 comets listed, no less than 21 have had their orbits changed in the elliptical direction as a result of computing the perturbations backwards in time in this way. In two cases small hyperbolic values of $1/a$ remain, but these are without significance in view of the mean error of the original values, and would certainly be reduced still more if the perturbations of Uranus and Neptune were included. We can say, therefore, that these investigations lead us to believe that all comets are members of the solar system, travelling in elliptical orbits with the Sun at one focus.

The number of comets

The number of such comets, on any reasonable assumption, must be enormous. Those which occur in our lists are merely the comets which happen to have been seen from the Earth—those comets, in fact, which have suitable values of q. Yet there is clearly no limitation on the size of such orbits, or on the value of q, and even if we cannot assume the uniform distribution of perihelia mentioned above, yet we must certainly envisage a vast number of comets which we shall never see, unless considerable perturbations bring them within a suitable distance of the Earth. Many of the comets which do approach the Earth undoubtedly escape detection, but we may take as a reasonable figure a total of 300 new comets every

century. If then we agree with Crommelin (1929) in estimating the average period of these comets at 40,000 years, we have a figure of 120,000 comets as the minimum number which revolve about the Sun. Comets are therefore the most numerous of all the bodies which form the solar system, and their orbits may be visualised stretching out into space, perhaps a third of the way to the nearest star.

The origin of comets

The origin of this great family of comets has engaged the attention of numerous workers in the course of the last two centuries. Since Laplace first suggested that comets have their origin in an interstellar cloud captured by the Sun, this particular theory has been studied from many points of view. The main objection to such an idea lies in the complete absence of any marked hyperbolic orbits, but in the most recent form of this theory, the difficulty seems to have been overcome. Lyttleton (1948) envisages a process of accretion of cosmic dust within the solar system itself, and develops his hypothesis to cover many of the difficulties which have been stumbling blocks to this theory in the past.

Nevertheless, the accretion theory implies different origins for the planets and comets, although there are a great number of similarities between the two classes of bodies, and the constitution and behaviour of meteorites form a natural link between the two. Lyttleton's scheme, therefore, is diametrically opposed to those theories which consider comets and planets to have had a common origin. The original suggestions due to Lagrange, that comets arose from the explosion of a planet, or from eruptions from the surface of a planet such as Jupiter, have always had considerable support. A critical review of these theories has now been given by van Woerkom (1948) who finds that the perturbations produced by Jupiter are of paramount importance in determining the distribution of the axes of long-period comets. The eruption theory does not give an adequate explanation of the distribution of inclinations in these large orbits, although it does account in a satisfactory manner for the characteristics of short-period comets. On balance, the capture theory of the origin of short-period comets would seem more plausible, and there is no escape from the conclusion that comets have always been members of the solar system.

This analysis of van Woerkom has been extended by Oort (1950) to a consideration of the structure of the entire system of comets which surround the Sun, and reasons are given for believing that " new " comets enter the region of visibility from distances of the order of 50,000 to 150,000 A.U.—distances which are comparable with that of the nearer

stars. In this vast sphere the number of comets is of the order of 10^{11}, and their distributions are determined not only by the perturbations of Jupiter, but also by those of the stars. Oort thus finds a natural explanation of the origin as well as of the distribution of comets, and suggests that both comets and meteorites are minor planets which have escaped from the ring of asteroids, and have subsequently settled into huge stable orbits under the influence of the planetary and stellar perturbations. It seems likely that future research will attach greater importance to the study of " new " comets, and of the perturbations by planets and stars which these comets have suffered in the past.

BIBLIOGRAPHY

A. CATALOGUES

FLAMMARION, C., *Annuaire Astronomique Camille Flammarion* (Paris).

CROMMELIN, A. C. D., 1925, *Catalogue of Comets, Memoirs Brit. Astr. Assocn.*, **26**, part 2.

CROMMELIN, A. C. D., 1932, *Continuation of Comet Catalogue, Memoirs Brit. Astr. Assocn.*, **30**, part 1.

GALLE, J. G., 1894, *Verzeichniss der Elemente der bischer berechneten Cometenbahnen* (Leipzig : Engelmann).

YAMAMOTO, A. S., 1936, *Preliminary General Catalogue of Comets, Pub. Kwasan Obs.*, I, No. 4.

B. BOOKS and LECTURES

CHAMBERS, G. F., 1909, *The Story of the Comets*, Ch. XV (Oxford : Clarendon).

KOPFF, A., 1929, *Kometen und Meteore, Handbuch der Astrophysik*, Band IV, 426 (Berlin : Springer).

OORT, J. H., 1951, *The Origin and Development of Comets* (The Halley Lecture) *Observatory*, **71**, 129.

VINTER-HANSEN, J., 1944, *The Orbits of Comets, Pop. Astr.* **52**, 370.

WATSON, FLETCHER, 1941, *Between the Planets* (Philadelphia, Blakiston).

C. REFERENCES

CROMMELIN, A. C. D., 1929, Article *Comets* in *Encyclopedia Britannica*, 14th edition.

FAYET, G., 1911, *Bull. Astr.*, **28**, 145.

LYTTLETON, R. A., 1948, *Mon. Not. Roy. Astr. Soc.*, **108**, 465.

OORT, J. H., 1950, *Bull. Astr. Inst. Netherlands*, XI, 91 (in English).

PICKERING, W. H., 1911, *Harvard Annals*, **61**, part 3.

PROCTER, R. A., 1884, *Knowledge*, VI, 126.

RUSSELL, H. N., 1920, *Astr. J.*, **33**, 49.

SINDING, E., 1948, *Pub. Copenhagen Obs.*, Nr. 146 (in English).

STRÖMGREN, E., 1914, *Pub. Copenhagen Obs.*, Nr. 19.

STRÖMGREN, E., 1947, *Pub. Copenhagen Obs.*, Nr. 144.

WOERKOM, A. J. J. van, 1948, *Bull. Astr. Inst. Netherlands*, X, 445 (in English).

CHAPTER IV

Perturbations

The cometary orbits which have been discussed in the previous chapters have all been osculating orbits, in which no allowance has been made for the disturbances caused by the major planets. Since every mass in the universe attracts every other mass in accordance with the law of gravitation, it is evident that no comet can exactly follow one of these conic sections, but is constantly being disturbed. The orbit which *is* followed may be regarded at any instant as an ellipse, but it is a variable ellipse, and different orbital constants must be given to it for different dates. If we consider first of all the type of perturbation which is most common, namely a moderately close approach to Jupiter, we gain an insight into the behaviour of comets at their various returns to perihelion, the effect of Jupiter being more serious, owing to its great mass, than that of any other planet.

The force of attraction is given by $F = GMm/r^2$, and if we represent the attractive force of Jupiter at unit distance by 1000, the values for the other members of the solar system at the same distance are as follows :

Sun	1047350		
Mercury	0·15	*Jupiter*	1000·00
Venus	2·58	*Saturn*	299·24
Earth	3·17	*Uranus*	45·80
Mars	0·34	*Neptune*	54·23

It will be seen that the main disturbances in the motion of comets are always caused by Jupiter and Saturn, except in the unusual case of a very close approach to one of the other planets. The main effect of an attraction of this kind is to cause an acceleration of the motion of the comet along the line comet-planet, but this does not mean that the comet will actually move in this direction. The acceleration along this line must be compounded with that along the line comet-Sun, and as a result there may arise various changes in the orbit. To take a simple case in illustration, a tangential force applied to the comet in the direction of its motion will tend to increase its velocity. Since this is given by

$$V^2 = k^2 (2/r - 1/a)$$

it is evident that the velocity at a given distance r from the Sun can only increase if the value of a increases, and it follows that a tangential force

of this kind will cause an increase in the size of the orbit. It is possible to examine the other effects of perturbations in a similar manner (Moulton, 1902, Chapter VIII), but it is perhaps more satisfactory to quote the actual formulae that may be employed.

Equations of motion

The equations of motion given by (50) may be amended to include the effect of an attracting body of mass m_1, whose co-ordinates are x_1, y_1, z_1, and whose distance from the comet is ρ. Whatever the system of co-ordinates employed,

$$\frac{d^2x}{dt^2} = -k^2 \sum m_1 \frac{x_1 - x}{\rho^3} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

but it is naturally more convenient to employ heliocentric co-ordinates in computing the perturbations. Equations (56) express the motion of the comet with reference to the centre of mass of the whole system, but similar equations may be written for the motion of the Sun, and the difference between the two equations gives the more convenient form :

$$\frac{d^2x}{dt^2} + k^2(1+m)\frac{x}{r^3} = k^2 m_1\left(\frac{x_1 - x}{\rho^3} - \frac{x_1}{r_1^3}\right) \qquad \ldots\ldots\ldots\ldots (57)$$

where the co-ordinates are now heliocentric, and r and r_1 are the radii vectores of comet and disturbing body respectively. This equation, with the corresponding ones in y and z, may be used to compute accurately the perturbations of the comet, as in the Cowell method, or may be used in other forms to obtain approximate results. The details of these methods are described in the standard text-books (e.g., Stracke, 1929), but the general principles may be understood by writing (57) in the form

$$\frac{d^2x}{dt^2} + k^2(1+m)\frac{x}{r^3} = m_1 \cdot \frac{dR}{dx} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (58)$$

where m_1R is the *perturbative function*. Of the various methods that have been devised to deal with this problem, the earlier ones all computed the value of the perturbative function or of quantities derived from it, and the most important of these methods is Encke's, which is still of considerable value in certain cases. Where strict accuracy is not required, it is more convenient to determine the *variation of elements* rather than the variation of co-ordinates. The problem is then greatly simplified by taking a new set of axes, which refer to the plane of the comet's orbit, but move with the comet ; the X axis is directed along the radius vector from Sun to comet, the Y axis in the plane of the orbit, to a point 90° in front of the comet, and the Z axis perpendicular to the orbit plane. It will be seen that the co-ordinates of the comet in this system are r, O, O, and the

equations corresponding to (57) become simplified. Three quantities **S**, **T**, **W** are then computed from these equations, and are to be regarded as the components of the attracting force measured along the three axes defined above. The effect of these forces on the orbital elements is readily obtained from a consideration of the changes they produce in $r\frac{dv}{dt}$ and in $\frac{dr}{dt}$; and the differentiation of the equations connecting the various elements (Chapter 2) leads to the following results :

$$
\begin{aligned}
w\frac{d\Omega}{dt} &= r\sin(\omega+v)\operatorname{cosec} i\,.\,\mathbf{W}\\
w\frac{di}{dt} &= r\cos(\omega+v)\,\mathbf{W}\\
w\frac{d\phi}{dt} &= a\cos\phi\sin v\,\mathbf{S} + a\cos\phi(\cos E+\cos v)\,\mathbf{T}\\
w\frac{d\pi}{dt} &= -\frac{p}{e}\cos v\,\mathbf{S} + \frac{p+r}{e}\sin v\,\mathbf{T} + 2\sin^2\tfrac{1}{2}i\,.\,\frac{d\Omega}{dt}\\
w^2\frac{dn}{dt} &= -\frac{3kw}{\sqrt{a}}\left(e\sin v\,\mathbf{S} + \frac{p}{r}\mathbf{T}\right)\\
w\frac{dM_0}{dt} &= (p\cos v\cot\phi - 2r\cos\phi)\,\mathbf{S} - \cot\phi\sin v(p+r)\,\mathbf{T}
\end{aligned}
$$

where w is the interval in days. ....(59)

Qualitative effects

From the qualitative point of view, the changes that take place in an orbit may be deduced from equations (59) without difficulty. Thus an orthogonal component **W** is, in practice, always caused by the major planets, which lie in or near the ecliptic. In direct motion, **W** will be negative as the comet moves forward from the ascending node, i.e., as $\omega + v$ increases from 0° to 180° ; and **W** will be positive in the remaining part of the orbit. The change in the node given by the first equation of (59) is thus always negative, and it follows that the effect of planetary attractions on a comet is always to cause the node of the comet's orbit to retrograde if the motion is direct, or to advance if the motion is retrograde. This is a common and very noticeable effect in the case of comets, and is of great importance in dealing with the orbits of meteor streams.

In the fifth equation of (59) the bracket on the right hand side may be shown to contain the resolved part of the perturbative force along the tangent, and the decrease in n (corresponding to an increase in a) is thus expressed mathematically. The other effects on the elements are summarised in Table 11, which is to be used in conjunction with Fig. 18.

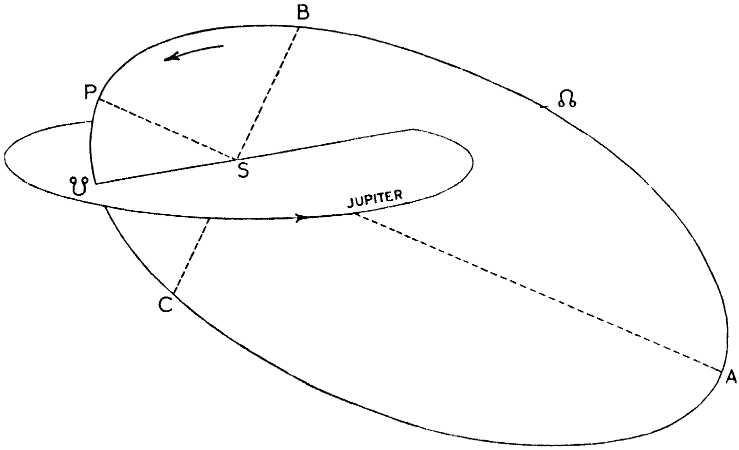


FIG. 18.—Illustrating perturbations (see Table 11).

Methods of calculation

Since the expressions in (59) give the rate of change of the elements at a given time, and this rate is continually changing, the total effect can only be obtained by a process of integration. The method is simplified by computing the values of these expressions at equal intervals of time (or of mean anomaly), since it is possible to use precomputed tables for the evaluation of many of the required quantities (Crommelin, 1928 ; Stracke,

TABLE 11

Perturbations

Node. Affected only by **W**, which changes sign at ☊. Hence the node always retrogrades in direct motion.

Inclination. Affected only by **W**. Has maximum positive value at ☊ and maximum negative value at ☋.

Major axis. An attraction +**T** always increases the major axis. +**S** has zero effect at perihelion and aphelion, increases the major axis in the region PCA and decreases it in the second half ABP.

Line of Apsides. +**S** causes the line PA to move forward while comet is in the region CAB, and backward during motion from B through P to C.
+**T** causes forward movement of the line PA when comet is in the region PCA ; backward in the region ABP.

Eccentricity. +**S** increases the eccentricity during motion from P through C to A, and decreases it in the region ABP.
+**T** has the maximum positive effect at P and the maximum negative effect at A ; and the effect is zero at the two points where $r = b$.

1929). A simple summation of the changes at each step is usually sufficient to give the total effect. The method is approximate only, because it computes the perturbations from the data for the undisturbed orbit. This difficulty leads to errors which may be almost completely eliminated

by occasionally *rectifying* the orbit, computing new elements and continuing the process with these improved figures. It would be more correct to rectify at each step, and this has actually been achieved in a modification recently introduced by Merton (1950) which is both rapid and sufficiently accurate for ordinary purposes. It is of interest to notice that a method has been suggested (Herrick, 1948) for the use of this method on electronic calculating machines.

Where the highest accuracy is required, the Encke method may be used (although this also entails rectification), or the equations (57) may be integrated directly by the Cowell method. Both methods give accurate values for the actual attractions, which are quantities of the second order, and the necessity for greater accuracy means that the integrations can no longer be carried out by simple summation, but use must be made of the formulae of the calculus of finite differences. The whole process of step-by-step calculation is known as *mechanical quadrature* and, apart from the necessity of accurate integration, its general principles may be illustrated by means of the familiar example of a body allowed to fall freely in the earth's gravitational field. In this case the acceleration may be taken as 32 ft./sec.^{2}, and this is entered in the third column of Table 12a to represent the acceleration at the beginning of each second. Column v gives the velocity at the middle of each time-interval, and since the velocity was initially zero, it must be 16 ft./sec. at the first mid-interval. With this figure as the first value in column v, the remaining velocities are obtained by simple addition. In the same way, the distances given in column s are

TABLE 12

Illustrating Mechanical Quadrature.

Time in seconds	12a			12b			12c		
	s	v	a	s	v	a	s	v	a
0	0		32	0		32	0		32
		16			16			16	
1	16		32	16		32	16		32
		48			48			48	
2	64		32	64		35	64		35
		80			83			83	
3	144		32	147		32	147		35
		112			115			118	
4	256		32	262		32	265		35
		144			147			153	
5	400		32	409		32	418		35
		176			179			188	
6	576		32	588		32	606		35

s = space described in feet. v = velocity in ft./sec. a = acceleration in ft./sec.^{2}

formed by summation, beginning with zero as the initial value. This process of mechanical integration corresponds to the solution of the differential equation $\frac{d^2x}{dt^2} = g$.

Simple as this scheme is, it illustrates the many peculiarities of the effect of perturbations on a comet. In such a case the acceleration is not constant, but is continuously variable, and each addition to the acceleration will persist in the velocity of the comet, and will accumulate in the distance travelled. This is illustrated in Table 12b in which an increase in the acceleration of the falling body occurs in the third second. It will be seen that the additional 3 units remain in the velocity column, and accumulate with time in the figures showing the distance described. If the increase in the acceleration is more long-lived, the effect is correspondingly greater. In Table 12c the effect of a persistent addition of 3 units to the acceleration is shown ; in this case the increase in the velocity accumulates with time, while the effect on the distance is proportional to $\frac{1}{2}n(n + 1)$ where n is the number of intervals involved. The importance of this effect in the case of the movement of a comet is readily appreciated—every perturbation, however slight, has some effect on the motion and position of the comet in its orbit.

The process actually used is somewhat more complicated, since the accelerations are not constant and the perturbations are applied in three dimensions instead of only one. In its most accurate form, first used by Cowell in his work on the eighth satellite of Jupiter, and later (1910, with Crommelin) on Halley's comet, the three components of the acceleration given by (57) are computed and integrated separately. The scheme is commenced with the known positions and velocities on the date of osculation. These are given by (54) and their differentiated forms :

$$x' = \frac{wk}{r\sqrt{a}} (B_x \cos E - A_x \sin E)$$

$$y' = \frac{wk}{r\sqrt{a}} (B_y \cos E - A_y \sin E)$$

$$z' = \frac{wk}{r\sqrt{a}} (B_z \cos E - A_z \sin E) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(60)$$

The attractions are computed by (57), written in the form

$$f_x = \frac{d^2x}{dt^2} = -k^2w^2\frac{x}{r^3} + k^2w^2\sum m_1\left(\frac{x_1 - x}{\rho^3} - \frac{x_1}{r_1^3}\right) \quad ..(61)$$

and using all the planets whose attractions it is desired to include. It will be noticed that the interval w occurs in these accelerations in the form of its square, and the velocities given by (60) also contain the interval, although the co-ordinates given by (54) do not. The work is greatly simplified by the tables given in the two volumes of *Planetary Co-ordinates* which

should be consulted for further details. The scheme proceeds by simple summation, and at each step the differences f^{I} to f^{VI} are entered. These act as an important check on the accuracy of the computed attractions, and also enable a very good estimate to be made of the attractions at the next step.

TABLE 13

The Cowell scheme.

Date	${}^{\mathrm{II}}f$	${}^{\mathrm{I}}f$	f_x	f^{I}	f^{II}	f^{III}	f^{IV}	f^{V}	f^{VI}
1948 Jan. 10	+12696443		−37700		−409		−16		+1
		−1473869		+5255		+88		+6	
Feb. 19	11222574		32445		321		10		−5
		1506314		4934		78		1	
Mar. 30	9716260		27511		243		9		+1
		1533825		4691		69		2	
May 9	8182435		22820		174		7		+3
		1556644		4517		62		5	
June 18	6625791		18303		112		−2		−3
		1574947		4405		60		+2	
July 28	5050844		13898		−52		0		−2
		1588845		4353		60		0	
Sept. 6	3461999		9545		+8		0		+2
		1598390		4361		60		+2	
Oct. 16	1863609		5184		68		+2		+3
		1603574		4429		62		5	
Nov. 25	+260035		−755		130		7		−3
		1604329		4559		69		2	
1949 Jan. 4	−1344294		+3804		199		9		−1
		1600525		4758		78		1	
Feb. 13	2944819		8562		277		10		+7
		1591963		5035		88		8	
Mar. 25	4536782		13597		365		18		−5
		1578366		5400		106		3	
May 4	6115148		18997		471		21		+5
		1559369		5871		127		8	
June 13	7674517		24868		598		29		+4
		1534501		6469		156		12	
July 23	9209018		31337		754		41		+3
		1503164		7223		197		+15	
Sept. 1	10712182		38560		951		+56		*0*
		1464604		8174		+253		*15*	
Oct. 11	12176786		46734		+1204		*71*		
		1417870		+9378		*324*			
Nov. 20	13594656		+56112		*1528*				
		−1361758		*10906*					
Dec. 30	−14956414		*67018*						

Table 13 illustrates the appearance of the work, and consists of part of the X-scheme for Pons-Winnecke's Comet during the 1946–1952 revolution. The smooth running of the sixth differences show that the work is proceeding satisfactorily, and the value of f_x on December 30 is estimated by assuming a value of 0 for the next sixth difference. Simple addition

then gives the figures in italics, and the true value of x is computed from

$$x = {}^{\text{II}}f_x + \frac{1}{12}f_x - \frac{1}{240}f_x^{\text{II}} + \frac{31}{60480}f_x^{\text{IV}} - \quad \ldots\ldots\ldots\ldots(62)$$

The values of y and of z are computed at the same time from the parallel schemes, and thence $r^2 = x^2 + y^2 + z^2$.
An accurate value of the attractions is then computed from (61) using the tables in *Planetary Co-ordinates*, and in this case a value of 67023 was obtained. The difference of 5 units is negligible, since the estimate of f_x is required merely to find the true value of x in (62). The correct values are written in the three schemes, and the work proceeds in this way right round the orbit.

The work may be considerably shortened by increasing the interval from 10 to 20, 40 or even 80 days as the comet moves more slowly at a greater distance from the Sun, and the higher differences become smaller. As the comet approaches the Sun once more, or passes close to a planet, it becomes necessary to decrease the interval again ; and if the approach to a planet becomes very close it is possible to invert the whole calculation so that the planet becomes the origin and the Sun is regarded as the disturbing body. By making the interval sufficiently short, the method affords complete accuracy, and with modern calculating machines it is not unduly laborious. It is usually considered that an interval of 10 days represents a convenient lower limit for use with this method, and this means in practice that the Cowell scheme is not usually adopted when the comet is at such a distance from the Sun that r^2 is less than 2. Under such circumstances the Encke method is used instead. This computes only the last term of equation (61), that is the deviations of the comet from the undisturbed orbit ; and although each step takes longer to do than the corresponding Cowell step, a 20-day interval usually suffices.

Details of the Encke method are given in most textbooks, while for the Cowell method the more modern books should be consulted. Valuable discussions of the Cowell method are given by Innes (1926) and Jackson (1924). Examples of both methods are given in the volumes of *Planetary Co-ordinates*.

Examples of perturbed orbits

The total effect of these attractions on the orbit of the comet will naturally depend on the conditions. The problem must be visualised from the standpoint of the integration table, for the total perturbations will depend, not only on the severity of the attraction due to a close approach to a major planet, but also on the time during which that attraction lasts. A prolonged effect may well have just as serious results as a much closer

approach for a shorter period. Thus a comet which remains in the vicinity of Jupiter for a long period (as may happen near aphelion in the case of a short-period comet) will suffer severe perturbations. The effect is clearly lessened if the comet's orbit is highly inclined to the ecliptic, in which case severe perturbations can only arise in the case of a close approach at the node. In the extreme case, where the comet has retrograde motion, the perturbations are necessarily much smaller, since comet and planet are moving past each other in opposite directions.

The general nature of the changes which take place in a comet's orbit may be illustrated by reference to P/Pons-Winnecke. The orbit of this comet has its aphelion close to the orbit of Jupiter, and the period is approximately half that of the giant planet. At alternate revolutions, the comet makes a close approach to Jupiter at the ascending node, and the resulting perturbations last for a considerable time. In the following table (Table 14) of approximate elements of the comet, the changes that have taken place in the orbit are clearly shown, and the total result has been to make the orbit larger, more nearly circular and of greater inclination. The regression of the nodes is also noticeable, while the increasing size of the orbit has altered the distance of nearest approach to the Earth's orbit, as shown in the column headed C–E. This comet gave rise to a meteor shower in 1916. The general features of the changing orbit are shown in Fig. 19.

TABLE 14

Changes in the orbit of P/Pons-Winnecke.

Date	☊	i	e	q	C – E
1858	113°·5	10°·8	0·755	0·769	− 0·231
1886	104·1	14·5	0·726	0·885	− 0·128
1915	99·4	18·3	0·701	0·972	− 0·041
1921	98·1	18·9	0·678	1·041	+ 0·030
1939	96·8	20·1	0·670	1·102	+ 0·092
1945	94·4	21·7	0·654	1·160	+ 0·150

In the more usual type of orbit there is no such correlation between the periods of the comet and of the disturbing planet, and the effects are smaller. In general it may be said that in order seriously to alter the orbit of a comet, its approach or series of approaches to a planet must be very close indeed. There have been a few cases in which the orbit of a comet has been altered very noticeably by the action of Jupiter, and the behaviour of Lexell's comet may be recalled in this connection. Discovered in 1770 by Messier, the comet came within 1½ million miles of the

Earth, and at one stage had a visible coma more than two degrees in diameter. The orbit, which was found to have a period of 5·6 years, was investigated by Lexell of St. Petersburg, who showed that the comet had made a very close approach to Jupiter in May 1767, when its orbit had been changed from a much larger ellipse to its present shape. The comet was never seen again, owing to a second close approach to Jupiter in 1779. Lexell's work, which led to the association of his name with the comet, was later confirmed by Burckhardt and Leverrier.

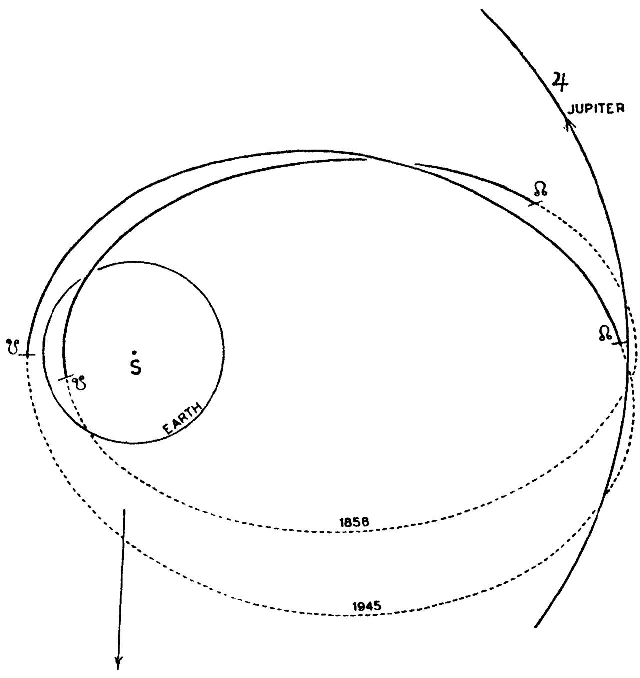


FIG. 19.—The orbit of P/Pons-Winnecke.

Considerable changes in the orientation of a comet's orbit are more common, and may be illustrated by the cases of P/Grigg-Skjellerup and P/Wolf (1):

	P/Grigg-Skjellerup			*P/Wolf* (1)	
	1902	1922		Before 1922	After 1922
i	8°·7	17°·5	P	6·82 yrs.	8·31 yrs.
q	0·75	0·89	q	1·58	2·47
e	0·739	0·695	e	0·56	0·40

P/Wolf (1) has been the object of intensive study by Kamienski (1948) who has given reasons for believing that the comet is slowly fading.

The actual disruption of a comet has been witnessed on a number of occasions, but such an occurrence has not always been associated with close approaches to Jupiter or any other planet. P/Brooks (2), 1889 V, was shown by Chandler to have had its period changed from 29 years to 7 years by a close approach to Jupiter in 1886. During its appearance in 1889 it threw off four fragments, two of which were visible for some time as miniature comets, one of them actually becoming brighter than the parent body. The comet was seen again in 1896 and 1903, and in 1921 it made another close approach to Jupiter, as a result of which the longitudes ω and Ω were greatly altered, although the remaining elements were little changed :

	1903	1932
ω	$343^\circ{\cdot}6$	$195^\circ{\cdot}8$
Ω	18·1	177·5
i	6·1	5·5

It will be seen that the longitude of perihelion ($= \omega + \Omega$) remained practically unchanged, but the whole orbit had altered its inclination about the line of nodes through some 12°.

The disruption of Biela's comet is referred to in a later chapter, while other cases in which a comet has been seen to divide into two or more parts include the great comet of 1882 III, and comets 1860 I and 1916 I (Taylor's comet). There is no evidence in these cases that the disruptions were caused by planetary perturbations.

Tisserand's Criterion

Although the elements of a comet may undergo considerable changes from one apparition to the next, it is possible to show the identity of the two computed sets of elements by means of the criterion derived by Tisserand (1896, Tome IV, p. 203). If the elements a, p and i refer to the comet, and a_1 is the semi-axis major of the disturbing planet (always taken to be Jupiter) and r_1 the mean radius vector of that planet during the period of disturbance, then

$$\frac{1}{a} + \frac{2\sqrt{a_1}\sqrt{p}\cos i}{r_1^2}$$

remains invariable, and in particular, it will have the same value for both apparitions of the comet. The formula is greatly simplified by some writers, who refer i to the plane of the ecliptic instead of the plane of Jupiter's orbit, while r_1 is also taken as equal to a_1. The invariant is necessarily only an approximation, but it has been applied with success in a number of doubtful cases.

BIBLIOGRAPHY

A. BOOKS

BAUSCHINGER, J., 1928, *Die Bahnbestimmung der Himmelskörper*, 2nd edition (Leipzig : Engelmann).

HERGET, P., 1948, *The Computation of Orbits* (Cincinnati : printed privately).

MOULTON, F. R., 1902, *Celestial Mechanics* (New York : Macmillan).

OPPOLZER, T. R. v., 1882, *Bahnbestimmung der Cometen und Planeten* (Leipzig : Engelmann).

STRACKE, G., 1929, *Bahnbestimmung der Planeten und Kometen* (Berlin : Springer).

TISSERAND, F. F., 1889–1896, *Traité de Mécanique Céleste*, 4 vols. (Paris : Gautier-Villars).

WATSON, J. C., 1900, *Theoretical Astronomy* (Philadelphia : Lippincott).

B. TABLES (*see also books listed above*)

BAUSCHINGER, J., 1901, *Tafeln zur Theoretischen Astronomie* (Leipzig : Engelmann).

CROMMELIN, A. C. D., 1928, *Tables for the computation of Perturbations, Memoirs Roy. Astr. Soc.*, **64**, 149.

PLANETARY CO-ORDINATES (1800–1940), 1933 (London : H.M. Stationery Office).

PLANETARY CO-ORDINATES (1940–1960), 1939 (London : H.M. Stationery Office).

C. REFERENCES

COWELL, P. H., and CROMMELIN, A. C. D., 1910, *Investigation of the motion of Halley's Comet* (Appendix to *Greenwich Observations* 1909. Edinburgh : H.M. Stationery Office).

HERRICK, S., 1948, *Pub. Astr. Soc. Pac.*, **60**, 321.

INNES, R. T. A., 1926, *Astr. Nach.*, Nr. 5463, Band 228.

JACKSON, J., 1924, *Mon. Not. Roy. Astr. Soc.*, **84**, 602.

KAMIENSKI, M., 1948, *Bull. Ac. Pol. Sci. et Lettres*, Ser. A., 1948, 19.

MERTON, G., 1949, *Mon. Not. Roy. Astr. Soc.*, **109**, 421.

CHAPTER V

Meteor Streams

It is a remarkable fact that our knowledge of the astronomical significance of meteors is of comparatively recent growth. Apart from speculations by Halley and others in the eighteenth century, the first definite evidence that meteors were not in any way comparable with other meteorological phenomena arose from an investigation of meteoric stones. Chladni in 1794 had given reasons for a belief in the celestial origin of the Pallas meteorite, but even Chladni's reputation was insufficient to win universal acceptance for the arguments he gave for this novel idea. Interest having been roused, however, evidence that stones of this kind had fallen from the sky was gathered from all quarters, and when, in 1803, a shower of stones fell at L'Aigle in Normandy, the French Academy deputed the physicist Biot to investigate. His report removed all doubt as to the origin of the stones, and the old records, dating back as far as 654 B.C., were substantiated.

The various showers of shooting stars that occurred on certain nights of the year seem to have attracted little attention until the night of November 12–13, 1833, when a veritable storm of meteors was seen in America. On this occasion, meteors of all sizes fell as thickly as snowflakes, and caused considerable alarm among superstitious people. To interested observers, however, it was clear that all the meteors appeared to radiate from one particular point in the sky, and this seems to be the first occasion on which the actual position of the *radiant* of a meteor shower was measured, although Humboldt had referred to the fact of a centre of radiation during his observations in 1799. The appearance was recognised as being due to perspective, the parallel lines of the meteor paths apparently vanishing to a point in the distance, as in the more familiar example of railway lines or telephone wires. In 1834 Olmsted (and Twining) suggested that the shower was caused by a cloud of particles through which the earth passed each November, and the idea of a periodic phenomenon was at last born. The very much weaker showers of 1832 and 1834 did not completely support this view, and Olbers seems to have been the first to suggest a 33-year period for these Leonid meteors.

The August Perseids, known traditionally as the "Tears of St. Lawrence," were shown to be periodic by Quetelet in 1836, and since the two phenomena could not both be due to a cloud of particles, Erman in

1839 suggested that the meteoric dust was arranged in a ring in each case. Twenty-five years later H. A. Newton of Yale investigated records of the November meteors, tracing the recurrence of the shower in a 33¼-year period from A.D. 902. Newton showed that the date of the shower had been delayed by one day in about 70 years, and suggested that this slow retardation of the date of the display might be used as a criterion to distinguish between the possible orbits of the meteoric dust. The arduous work that this involved was undertaken by J. C. Adams of Cambridge and he was able to show that of the five orbits which Newton had postulated, only the largest, that with a period of 33 years, was capable of satisfying the conditions.

Newton predicted a recurrence of the Leonid shower in 1866, and a fine shower was seen at the expected time. At the end of that year came the unexpected announcement by Schiaparelli that the August meteors move in the same orbit as the comet of 1862. This was rapidly followed by the identification by Leverrier and Peters of the orbit of the November Leonids with that of Tempel's comet of 1866, confirmed independently by Schiaparelli ; while Weiss showed the agreement between the Lyrids and the comet of 1861, and between Biela's Comet and the meteors of November 18.

The idea that meteors and comets were closely connected had been suggested by Kirkwood in 1861 :

> " May not our periodic meteors be the debris of ancient but now disintegrated comets, whose matter has become distributed round their orbits ? "

The question was brought to a head, however, by the behaviour of Biela's comet, a faint object which was discovered by Biela in 1826 and shown to be the same comet as those of 1772 and 1805. It was seen again in 1832 and 1846, but on this last occasion, the comet divided into two parts. The two comets travelled side by side in space, and were seen again at the 1852 return, having separated somewhat in the interval, but they have not been seen again. In 1872, when the earth passed close to the orbit of the comet, there was a magnificent display of shooting stars. It is this event, more than any other, which has given rise to the view that a meteor stream is the debris of a comet, but it must be emphasised that if this is the case, the comet must inevitably be " ancient but now disintegrated," in the words of Kirkwood. The Andromedids which are associated with Biela's comet had given displays radiating from a point near the star γ Andromeda in 1741, 1798, 1830, 1838 and 1847. The connection between the comet and these showers was shown independently

by Weiss, d'Arrest and Galle, and a fine shower was seen in 1867 in accordance with expectations. The great display of 1872 was repeated in 1885, and lent further support to the generally accepted view that Biela's comet had completely disintegrated, leaving a stream of meteoric particles in its wake. This is indeed the impression that is created by some modern writers, but there was no such wide generalisation among those who were more closely in touch with these events.

> " Nor did the meteors of November 27 directly replace the vanished comet. They too must have separated from it at a much earlier stage of its history." (Clerke, 1885.)

The fact that showers are associated with comets is too often distorted into meaning that the stream causing the shower is composed of the debris of that comet ; yet the Andromedids were seen before Biela's Comet was discovered, and the Leonids, Lyrids and Perseids have been recorded for hundreds of years, although their associated comets were first seen in the nineteenth century. There is no *direct* evidence that meteors are formed by the disruption of a comet, indeed all the available facts suggest that the disruption of a comet gives two or more comets. It is true that many comets are believed to lose mass after repeated returns to perihelion, but meteor showers are not always associated with comets which have made many returns. The differences that exist between the behaviour of the head of a comet and the diffuse mass of a meteor stream make it very difficult to visualise any process by which they may be transformed from one into the other. A few speculative ideas have been put forward (Corlin, 1938) by which bodies such as meteorites could be built up from smaller particles, but on the whole it is very much simpler to assume that comets disintegrate. The occurrence of such a process, however, can only be inferred—it has never been witnessed.

Visual observations

In the half-century that followed the recognition of the true nature of meteoric phenomena, the main facts of the recurrence of these displays were firmly established. Later workers, prominent among whom was W. F. Denning, adopted improved standards of accuracy, both in observation and recording of results, and it soon became apparent that the number of showers which occur during the year was far greater than had been imagined. Denning, in fact, was of the opinion that there are more than 50 showers in play on any and every night of the year ; and in 1899 he published his *General Catalogue of the Radiant Points of Meteoric Showers*, which contains no less than 4,367 radiants grouped about 278

centres of radiation. Denning's *Catalogue* still serves as an important guide to the distribution of meteoric showers ; but it must be used with discretion, since many of the radiants which it contains are mean positions for the radiation over a considerable period of time. At a later epoch it was recognised that a radiant may have a complex structure, and may change its position, or even vanish and be replaced by other radiants in the course of a few hours ; and it became necessary to make definite rules as to the period of time during which a watch may be extended in order to establish the position of a radiant.

The early workers plotted the paths of the meteors which they observed on maps drawn on the gnomonic projection. The meteor paths being straight lines on such a map, the radiant is determined by projecting the paths in a backward direction to the point of intersection. To guard against the many spurious points of intersection that may be found when many lines are drawn on the map, rules were formulated defining the number of intersections within a given area and their closeness of fit. The method leads to the determination of radiants from the observations of one observer, but it is also possible to find the radiant of an isolated meteor if this is seen by two or more observers. The parallactic displacement of the meteor, due to the different positions of the observers, leads not only to the position of the radiant, but also to the true heights and speed of the meteor. The method of duplicate or multiple observation (Prentice, 1932) is of proven value in the study of the minor streams.

In more recent times considerable improvements have resulted on the observational side, in visual work, from the technique of Prentice (1945) who abandons the map and plots the path of the meteor by means of angular distances and cross-bearings from known stars. The method is of superior accuracy (Porter, 1944), but entails a knowledge of the night sky which few observers possess. On the computational side, Davidson (1936) published a method of computing the true heights and paths of meteors within our atmosphere, by finding separately the points of intersection of each observation with the plane of the observations of the other observer. The method is free from assumptions, is capable of extension to least-squares solutions in the case of multiple observations (Porter, 1942), and has proved itself capable of facilitating the reduction of large numbers of observations.

The numbers of meteors

The number of meteors falling in a given time was early recognised as being a variable quantity. On the average, any one observer may see about ten meteors an hour, and taking into account the area of sky which is

visible from the place of observation, it is possible to estimate the number of meteors falling on the whole earth in one day. The best estimates make allowances for such varying factors as the apparent brightness of the meteors, and range from 10^6 to 10^8 meteors per day. The number of visible meteors is greatly increased at the times of the major showers, which are given in the following list (Prentice, 1948) :

TABLE 15

The Major Showers.

Shower	Maximum	Radiant R.A.	Radiant Dec.	Normal hourly rate
Quadrantids	Jan. 3	230°	$+52^\circ$	30–40
Lyrids	Apr. 21	270	+ 33	7–10
Perseids	Aug. 10–13	47	+ 58	40–60
Orionids	Oct. 20–23	96	+ 15	10–20
Leonids	Nov. 16–17	152	+ 22	10–15
Geminids	Dec. 11–13	113	+ 32	60

The showers are named after the constellation in which their radiant lies, or alternatively the name of the star nearest to this radiant is used as a basis, particularly in the case of minor showers. Exceptions are occasionally found, thus the Andromedids are also known as Bielids (from Biela's Comet), while the Giacobinids arise from Comet Giacobini-Zinner. These two showers have each given rise to great storms of shooting stars, a rare phenomenon which has otherwise been recorded only with the Leonids in 1833 and 1866.

Quite apart from the presence or absence of a shower, the hourly rate was soon recognised as a function of the time of night, and of the time of year. The early mathematical work on this subject was almost entirely due to Schiaparelli (1867), and most of the subsequent developments of this subject have been based on his pioneer work.

The influence of the apex

The diurnal and seasonal variations in the number of meteors is caused by the motion of the earth. The observed radiant of a meteor stream does not represent the true direction of approach of the meteors, but only the apparent direction. The direction of the Earth's motion at any moment is defined by the point in the sky towards which it is moving, and this point is known as the *apex of the Earth's way*. Owing to the elliptical nature of the Earth's orbit, the actual angular distance of the apex from the Sun

varies during the year, but can never differ from 90° by more than a degree. If L is the longitude of the Sun, and A that of the apex, then

$$A = L - 90^\circ + D \qquad \text{.............................(63)}$$

where

$$D = 0^\circ{\cdot}96 \sin[L - 102^\circ{\cdot}08 - 0^\circ{\cdot}0172\,(t - 1950)]$$

It is convenient in practice to refer all measurements to the same equinox of 1950·0, and the value of D in (63) and the tabulated values in Appendix 3 are all computed for this equinox.

The Earth's motion towards the apex is compounded with that of the meteors by the theorem of the triangle of velocities. Thus in Fig. 20 if

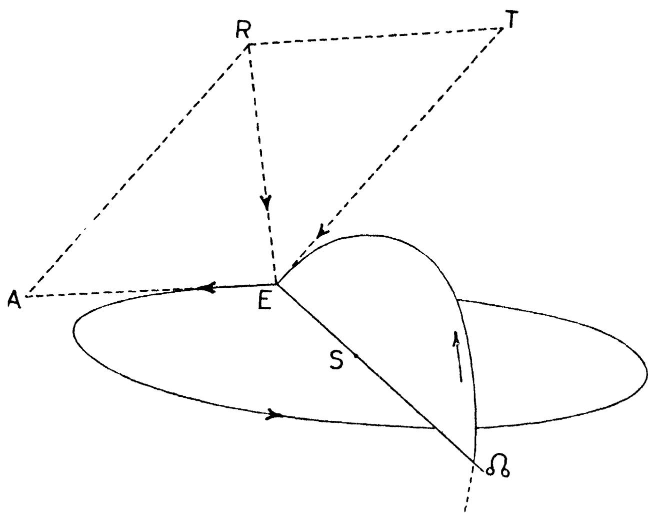


FIG. 20.—The apparent radiant of a meteor stream.

TE represents the velocity and true direction of motion of a meteor, and EA the velocity and direction of motion of the Earth, then the velocity and apparent motion of the meteor will be given by the diagonal RE. It is clear that the apparent radiant is always displaced in the sky towards the apex, so that there will be, in general, more apparent radiants in the hemisphere containing the apex than in the opposite hemisphere. It is this fact that accounts for the variation in the number of meteors. The apex, which is on the ecliptic and about 90° west of the Sun, will rise in the east about midnight, and will be at its greatest elevation south at about 6 a.m. Thus more meteors are to be expected after midnight than before it. Moreover, the elevation of the apex, which varies with the season, is at a maximum in northern latitudes in the early morning hours in autumn, because at this time the ecliptic lies north of the equator and rises steeply from the eastern horizon. For this reason, meteors are more

common in the autumn than at any other season, and in general, the diurnal and seasonal variation in the frequency of meteors is caused by a variation in the altitude of the apex.

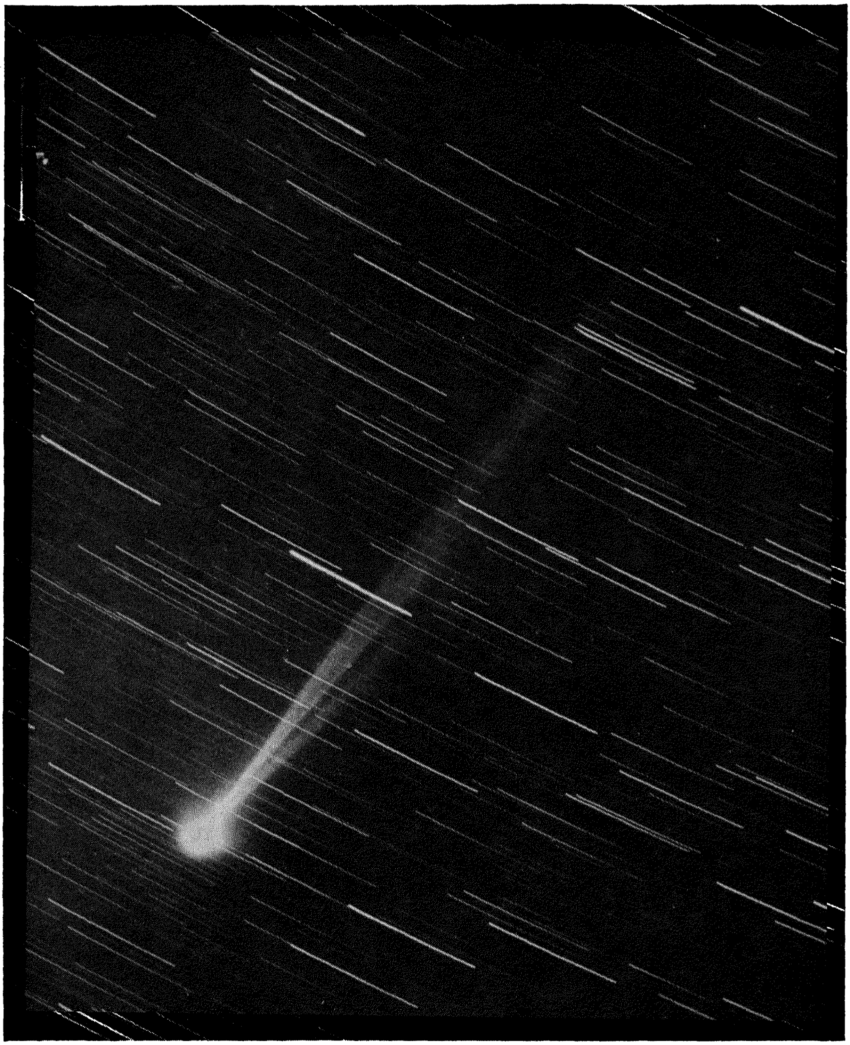
Schiaparelli was the first to make use of the observed frequencies of meteors to estimate the average velocities of these bodies. On the assumption of a uniform distribution of radiants over the celestial sphere, it is possible to deduce the theoretical variation of meteor frequencies (Davidson, 1914), and Schiaparelli showed that the observed frequencies were more in accordance with theory if the velocities of the majority of meteors were hyperbolic. It is interesting to reflect that these ideas, which are quite at variance with the assumption that meteors are derived from the debris of comets, have gained a very considerable following until quite recent times. It is the more unfortunate that all too little emphasis has been laid on the fact that the two ideas are quite inconsistent—if the majority of meteors have hyperbolic velocities, then the majority of meteoric particles cannot be derived from comets.

Modern methods

Attempts to improve on the naked eye observations had already been made more than half a century ago. By 1891, Elkin at Yale had commenced a series of observations of meteors, using equatorially mounted cameras at three stations (Olivier, 1937), and by 1899 the Yale camera had been fitted with rotating sectors with the object of securing meteor velocity measurements. The sectors were mounted on bicycle wheels driven by small electric motors, the current being derived from bichromate cells. Somewhat similar arrangements were used by Lindemann and Dobson (1922) in their photography of meteors with fixed cameras, used in the investigation of the upper atmosphere.

For nearly twenty years a regular programme of meteor photography has been undertaken at Harvard, and it is interesting to note that although photographic emulsions are faster, and optical equipment has greatly improved, the methods remain the same. The bicycle wheel of Elkin has been replaced by a smaller sector driven by a synchronous motor, but the battery of cameras mounted on an equatorial stand is still used. The rotating sector interrupts the trail at regular intervals (usually 1/20 second), and with a knowledge of the distance of the meteor, obtained from duplicate observations, such photographs are capable of giving velocities of high accuracy.

The number of photographs that can be obtained in this way is disappointingly small. A meteor which gives the same impression to the naked eye as would be given by the movement of a first magnitude star, cannot produce a similar effect on a photographic plate, and it is only the



COMET FINSLER, 1937 V

The coma and tail were noticeably unsymmetrical. Exposure 5 h. 21 m.

(*Blumbach, Norman Lockyer Observatory*)

brightest meteors that can be photographed. Thus, although of greatly superior accuracy, the photographic method must be judged for the present on the results of a comparatively small number of measured plates.

The radar methods which have been developed in recent years, although less accurate, are capable of giving data for large numbers of meteors. The early workers in this field used radio frequencies of the order of 20 Mc./s., but experience with Army radar equipment working on higher frequencies (70 Mc./s.) enabled Hey and Stewart in 1945 to establish the connection between meteors and the transient echoes that were recorded on the radar screens (Hey and Stewart, 1947). In the same year these workers were able to measure the position of the radiant of the δ-Aquarid stream, and also detected a shower in daylight hours. In 1946 the Giacobinid shower of October 10 provided conclusive evidence of the relation between meteors and the short-lived recorded echoes (Hey, 1946), which at the height of the shower attained the extraordinary rate of 168 per minute, the normal echo rate (no shower) being about 2 per hour (Lovell, Banwell and Clegg, 1947). A continuous systematic watch has been kept by Lovell at Manchester since that time, and the immediate results included the detection of the well-known night showers and the discovery of a remarkable system of daytime streams (Clegg, Hughes and Lovell, 1947).

The method of determining the radiant of a meteor stream by radio technique depends on the remarkable aspect-sensitivity of the reflections at these high frequencies. It was shown by Hey and Stewart (1947) that the radio echo from a given trail can only be observed if the trail crosses the beam at right angles. Utilising this fact, Clegg (1948) devised his system of determining radiants from one station only. A fixed beam is directed at a small elevation towards a particular aximuth, for example due east. No echoes are observed until the time of transit of the radiant, when the meteors will cross the beam at right angles. The appearance of the first long-range echoes gives the time of transit, from which the sidereal time and thence the R.A. of the radiant is determined. Subsequent echoes show a gradual decrease both in range and in numbers, and if the polar diagram of the aerial system is known, it is possible to compute a theoretical range-time distribution for comparison with the actual recorded results.

The declination of the radiant is found by making similar measurements in some other aximuth, the difference in the times of transit at the two aximuths being a function of the declination. Using this technique at Manchester, the radiant of the η-Aquarid shower was detected in the early morning of May 1, 1947, but the decreasing ranges of the Aquarids were

followed by a very pronounced peak at $10^h 40^m$. The activity of this new shower in Pisces rapidly increased after May 7, and was joined by a number of other showers whose activity certainly entitles them to be considered as major showers. Results of very great interest have been obtained from this confusing array of meteor streams, which continue each year to give such extraordinary displays from May until August, ceasing entirely in September. Among other results the new radar techniques has been used to investigate the reported return of the Bielids in 1947 (Clegg, Lovell and Prentice, 1948), and to confirm the observation in 1945–6 by Bečvář of a stream radiating from Ursa Minor (Clegg, Hughes and Lovell, 1948).

The structure of a meteor stream

Whatever may be the explanation of the origin of the meteor stream, there is no doubt that the resulting shower is caused by the passage of the earth through the swarm of particles. This can only occur when the earth is at, or near, a node of the meteoric orbit, but it does not follow that a shower must occur at each such nodal passage, that is, once in every fear. Indeed, this could only happen if the meteoric dust is spread out round the whole orbit. In other circumstances, as when the particles yorm a cluster at some particular point, then a shower is produced only when the earth intersects this cluster. If the period of the cluster is not commensurate with that of the earth, then very long periods may elapse before any shower is seen at all. The fact that the major showers give regular displays is thus a proof that the particles are distributed to a considerable extent in the orbit.

The actual distribution of the particles in such a cluster may be visualised from the known circumstances of some of the major showers. Taking the earth as approximately 12,700 kms. in diameter, and travelling at 25 km./sec., we see that it sweeps out a cylinder of space of volume $\frac{\pi}{4}(12700)^2.25.60.60.24$ cubic kms. in the course of a day. During this time, let us assume an average of 20 million meteors to fall on to the Earth. If their distance apart is d, the volume they occupy will be $20{,}000{,}000\ d^3$ cubic kms.

Equating these two quantities, we find the value of d to be about 260 km. In the case of the Perseids, where the hourly rate may be increased by a factor of ten, the distance is reduced to 120 km. Even in the case of unusual showers such as that of the Leonids in 1833, where the hourly rate rose to 35,000, the distance between the particles must have been of the order of 15 to 30 km. These distances depend, as may be seen above, on the cube root of the rate, and are always large. In no circumstances

can the packing of the particles approach that in the head of a comet. In fact, if we assume that a comet's head is made of the same materials as a meteor stream, we can only conclude that these materials are packed ten—or a hundred—thousand times more closely in the cometary nucleus than they are in the most condensed meteor stream.

Nor is this the only remarkable difference between the two bodies. For the meteoric particles show no evidence of an atmosphere of gases, and, moreover, they behave, not as one agglomeration of matter, but as a series of individual particles, each of which follows its own orbit, and suffers perturbations which differ from that of its neighbours. As a result, the mass is rapidly rendered diffuse, the matter being spread out in all directions. A condensed cluster thus becomes in the course of time a more attenuated stream, in which the meteoric dust is spread out in a non-uniform manner round the orbit. This lack of uniformity is apparent in the behaviour of the Leonids already referred to. Nothing could be farther from the truth than the " bicycle-tube " idea of a meteor stream. There is nothing uniform about such a structure ; rather is the stream to be imagined as a diffuse ring, throughout which numerous condensations and rarefactions are apparent, continuously changing position and size as they are affected by perturbations.

Nevertheless, such a structure could only be attained after long periods of time, covering many revolutions of the particles about the Sun. Thus the Perseids, which give a display every August with great regularity, are considered to be of much greater age than other showers, whose appearance is more variable. The structure of a much younger stream may be illustrated by the case of the Giacobinids. In the following table, the eccentricity and perihelion distance of the orbit of the comet (P/Giacobini-Zinner, Fig. 21) is given, together with the distance between the Earth's orbit and that of the comet at its node.

TABLE 16

Comet Giacobini-Zinner.

Year	e	q	Distance C – E	Earth at Node
1900	0·733	0·9319	– 0·0619	
1913	0·720	0·9759	– 0·0181	
1926	0·717	0·9947	+ 0·0005	70 days before comet : shower
1933	0·716	0·9994	+ 0·0054	80 days after comet : shower
1939	0·717	0·9955	+ 0·00125	136 days before comet : no shower
1946	0·717	0·9957	+ 0·00142	15 days after comet : shower

It will be seen that meteor showers occurred as a result of the passage of the earth through debris in front of the comet as well as behind it ; and the same fact has been noticed in the showers of the Andromedids from

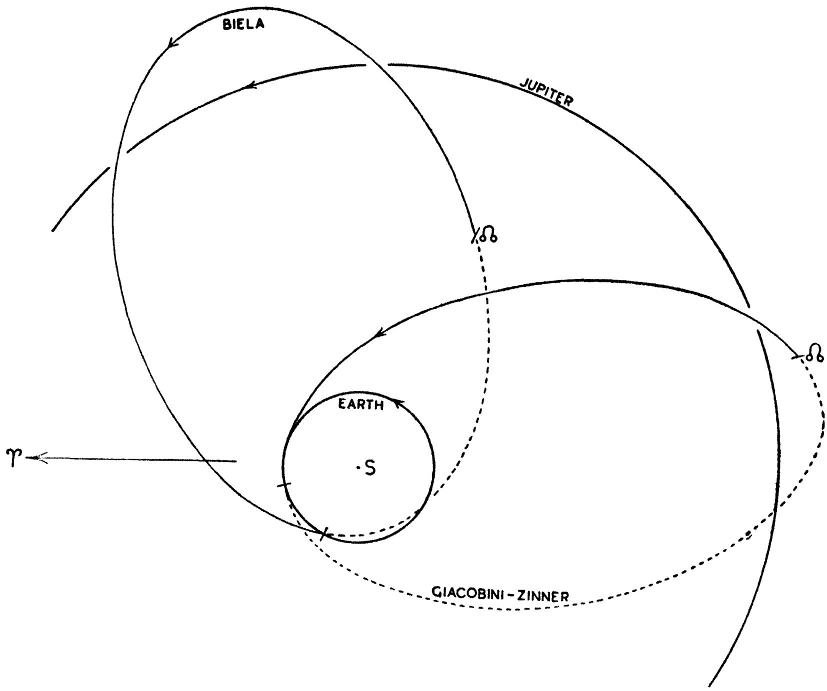


FIG. 21.—The orbits of P/Giacobini-Zinner and P/Biela.

Biela's comet in 1798, 1830 and 1838, when, on each occasion, the earth must have arrived at the comet's node before the comet itself. The prediction of a meteoric shower is clearly a difficult matter, since the failure of a stream to give a display at any nodal passage may be caused by the non-uniform distribution of the dust around the orbit, or to planetary perturbations. It must also be remembered that the stream may intersect the earth's orbit from the direction of the Sun, so that the resulting shower is present only in daylight hours.

Perturbations of a stream

The nature of the perturbations which have affected the particles of a meteor stream in the past must be governed by the same considerations that were given in the case of comets. In particular it is to be noted that the stability of the stream is likely to be greater if the motion is retrograde, or if the orbit is highly inclined, and to this fact is to be attributed the remarkable persistence of some of the major showers. The Perseids, for example, like the Leonids, travel in retrograde orbits ($i = 115°$ and $163°$ respectively), while the Lyrids ($i = 80°$) are highly inclined to the ecliptic. These are undoubtedly the oldest of the major showers, the Leonids having

been seen in November since A.D. 902, while there are records of the Perseids in August for over 1,200 years, and of the Lyrids in April for 2,500 years. The historical records of such showers show how little they have been affected by perturbations.

There is, however, one perturbative effect which is found to be present in all orbits. This is the changing position of the node which is caused by the orthogonal component W. As in the case of a comet (see page 57) there is always a regression of the node if the particles are moving direct, while the nodes advance if the motion is retrograde. This point is of particular importance in identifying a meteor shower with a comet, and was the basis of the laborious work of J. C. Adams on the orbit of the Leonids. In the same way, the Andromedids were identified with Biela's comet because they showed the same rapid regression of the nodes of one day every 7 years.

The gravitational effects which cause these perturbations are not the only forces which are called into play in determining the nature of a meteoric orbit. The impact of radiation pressure and the effect of relativity must also be considered, and the combined effect (now called the Poynting-Robertson effect) is of considerable importance in its action on meteoric dust, particularly in the immediate neighbourhood of the Sun. A small particle in the solar system is subject to the pressure of solar radiation, and the absorption of heat and its subsequent re-emission causes a retarding force on the moving particle. As a consequence of this the body moves in towards the Sun, travelling along a spiral path for 10^5 years or more before it ultimately falls into the Sun (Poynting, 1903). A more complete analysis by Robertson (1937) gives the equations of motion for such a particle, and includes the relativity effects. These equations have been used by Wyatt and Whipple (1950) to show that the well-known showers must have lost their smallest (and visually faintest) particles in times that are relatively short astronomically.

It is known that the neighbourhood of the Sun is far from being devoid of meteoric particles, and indeed the occurrence of the Zodiacal Light would indicate that the process of separation is still continuing. Nevertheless the investigation of these effects is valuable in giving a more precise picture of the manner in which a meteoric cluster is transformed into a broad diffuse ring of dust surrounding the Sun. The dimensions of such a ring may be roughly estimated in the case of an old shower such as the Perseids. This shower lasts for at least 12 days, during which time the earth has travelled some 30 million kms. The stream crosses the Earth's orbit at an angle of about 14°, so that the width of the stream must be about $3 \times 10^7 \sin 14° = 7{,}000{,}000$ kms. As it is unlikely that the Earth passes through the centre of the stream, this figure is a lower limit.

The width of this stream is such that the shower is visible over a long period. During the course of 24 hours the Earth has moved about one degree in longitude, and thus the angle at which the meteors enter the Earth's atmosphere is modified by this amount. The movement of such a long-enduring radiant by about one degree in longitude on successive nights is a further proof of the presence of a true meteor stream.

There are thus four criteria by which the identity of the orbits of a comet and of a meteor stream may be determined. These are :

(*a*) similarity of orbits, provided that the comet orbit can make a sufficiently close approach to the earth ;

(*b*) recurrence of the shower in a period identical with that of the comet ;

(*c*) regression or advance of the date at which the shower occurs, corresponding to the movement of the node of the comet ;

(*d*) daily movement of the true radiant in the case of a persistent stream.

Complete agreement in all four cases is rare, and few cases are known in which identity may be said to be established. In the following list, the dates and radiants of the associated showers have been computed from the original cometary data, and the nearest distance of approach of the cometary orbit is given in the last column.

TABLE 17

The Major Showers.

Shower	Comet	Date	Radiant	Distance C – E
Lyrids	1861 I	Apr. 21	271° + 34°	– 0·002
June Draconids	Pons-Winnecke	June 30	208 + 54	– 0·042
Perseids	1862 III	Aug. 11	45 + 58	+ 0·010
October Draconids	Giacobini-Zinner	Oct. 10	262 + 54	+ 0·004
Leonids	1866 I	Nov. 15	151 + 23	– 0·052
Andromedids	Biela	Nov. 30	23 + 44	– 0·018

It is interesting to note that the change in the nodes has inverted the order of the last two showers, the Andromedids now occurring on November 14 and the Leonids on November 16.

In addition to the showers given in this Table, there is a high probability that the Taurids are associated with Encke's comet (Whipple, 1940), while the May Aquarids have been identified with Halley's comet (Olivier, 1925 ; Hoffmeister, 1937). The idea of a common origin for a comet and its associated meteor shower is preferable to the assumption

of a direct connection. In the case of Encke's and Halley's comets, it is quite unlikely that the present orbits could give rise to the Taurids and May Aquarids. Both comets, having small inclinations, approach the Earth's orbit more closely at some distance from the node than at the node itself, but the distances at this point are quite large—Halley 0·15, Encke 0·19 unit. Some limit must be placed on the definition of a close approach, and the distances given in the table above would suggest that this limit is quite small.

BIBLIOGRAPHY

A. BOOKS

CLERKE, A. M., 1895, *History of Astronomy during the Nineteenth Century* (Edinburgh : A. and C. Black).

HOFFMEISTER, C., 1937, *Die Meteore* (Leipzig : Akad. Verlag.).

KIRKWOOD, D., 1867, *Meteoric Astronomy* (Philadelphia : Lippincott).

LOVELL, A. C. B., and CLEGG, J. A., 1952, *Radio Astronomy* (London : Chapman and Hall).

OLIVIER, C. P., 1925, *Meteors* (Baltimore : Williams and Wilkins).

SCHIAPARELLI, J. V., 1867, *Note e Riflessioni sulla theoria astronomica della stella cadenti* (Firenze : Stamperia) ; 1871, A German translation by Boguslawski : *Entwurf einer Astronomischen Theorie der Sternschnuppen* (Stettin : von der Nahmer).

WATSON, FLETCHER, 1941, *Between the Planets* (Philadelphia : Blakiston).

B. REPORTS AND LECTURES

BEER, A., 1940, *Meteors*, Observatory, **63**, 229.

HEY, J. S., 1946, *The Giacobinid Meteor Shower*, Nature, **159**, 119.

HEY, J. S., and STEWART, G. S., 1947, *Radar Observations of Meteors*, Proc. Phys. Soc., **59**, 858.

LOVELL, A. C. B., 1948, *Meteoric Ionisation, Phys. Soc. Rep. Prog. Phys.*, **11**, 415.

OLIVIER, C. P., 1929, Article *Meteor* in *Enc. Brit.*, 14th edition.

ÖPIK, E., 1940, *Meteors, Mon. Not. Roy. Astr. Soc.*, **100**, 315.

PANETH, F. A., 1940, *The Origin of Meteorites* (Halley Lecture) (Oxford : Univ. Press).

PORTER, J. G., 1948, *Comets and Meteor Streams, Phys. Soc. Rep. Prog. Phys.*, **11**, 402.

PRENTICE, J. P. M., 1948, *Meteors, Phys. Soc. Rep. Prog. Phys.*, **11**, 389.

WHIPPLE, F. L., 1949, *The Harvard Meteor Program, Sky and Telescope*, **8**, No. 4.

C. REFERENCES

ALMOND, M. and HAWKINS, G. S., 1950, *Brit. Astr. Assocn. J.*, **60**, 251.

CLEGG, J. A., 1948, *Phil. Mag.*, Ser. 7, **39**, 577.

CLEGG, J. A., 1948, *Brit. Astr. Assocn. J.*, **58**, 271.

CLEGG, J. A., HUGHES, V. A. and LOVELL, A. C. B., 1947, *Mon. Not. Roy. Astr. Soc.*, **107**, 369.

CLEGG, J. A, HUGHES, V. A. and LOVELL, A. C. B., 1948, *Brit. Astr. Assocn. J.*, **58**, 134.

CLEGG, J. A., LOVELL, A. C. B. and PRENTICE, J. P. M., 1949, *Brit. Astr. Assocn. J.*, **60**, 25.

CORLIN, A., 1938, *Zeit. für Astrophysik*, Bd. 15, 239 (in English).

DAVIDSON, M., 1914, *Brit. Astr. Assocn. J.*, **24**, 352 and 477.

DAVIDSON, M., 1936, *Brit. Astr. Assocn. J.*, **46**, 292.

DENNING, W. F., 1899, *Memoirs Roy. Astr. Soc.*, **53**, 203.

LINDEMANN, F. A., and DOBSON, G. M. B., 1922, *Proc. Roy. Soc.*, A, **102**, 411 and 1923, *Mon. Not. Roy. Astr. Soc.*, **83**, 163.

LOVELL, A. C. B., BANWELL, C. J., and CLEGG, J. A., 1947, *Mon. Not. Roy. Astr. Soc.*, **107**, 164.

LOVELL, A. C. B., and PRENTICE, J. P. M., 1948, *Brit. Astr. Assocn. J.*, **58**, 140.

OLIVIER, C. P., 1937, *Astr. J.*, **46**, 41.

PORTER, J. G., 1942, *Memoirs Brit. Astr. Assocn.*, **34**, part 4, 37.

PORTER, J. G., 1943, *Mon. Not. Roy. Astr. Soc.*, **103**, 134.

PORTER, J. G., 1944, *Mon. Not. Roy. Astr. Soc.*, **104**, 257.

POYNTING, J. H., 1903, *Phil. Trans.*, A, **202**, 525.

PRENTICE, J. P. M., 1932, *Brit. Astr. Assocn. J.*, **42**, 245.

PRENTICE, J. P. M., 1945, *Brit. Astr. Assocn. Handbook*, 1945, 36.

PRENTICE, J. P. M., LOVELL, A. C. B., and BANWELL, C. J., 1947, *Mon. Not. Roy. Astr. Soc.*, **107**, 155.

ROBERTSON, H. P., 1937, *Mon. Not. Roy. Astr. Soc.*, **97**, 423.

WHIPPLE, F. L., 1940, *Proc. Am. Phil. Soc.*, **83**, 711.

WYATT, S. P., and WHIPPLE, F. L., 1950, *Astrophys. J.*, **111**, 134 = *Harvard Reprint* 335.

CHAPTER VI

Meteor Orbits

The measurement of the apparent radiant of a meteor stream is the first step in a long series of calculations that lead to a knowledge of the orbit of that stream ; and this orbit can be determined with complete accuracy only if the velocity of the meteors is known. The true radiant, which gives the actual direction of approach of the stream, is obtained from the apparent radiant by applying corrections (*a*) for the Earth's attraction and (*b*) for the aberrational effect due to the Earth's rotation, and it is then possible (*c*) to compute from this corrected radiant the position of the true radiant. All three steps involve a knowledge of the velocity of the meteors, and since more than one velocity is quoted in this problem, it is necessary first to define these in detail. The following symbols will be used :

V is the actual velocity of the meteor in its orbit at the time of meeting the Earth. It is given by equation (30) and is represented in magnitude and direction by TE (Fig. 20).

V_1 is the apparent velocity of the meteor, compounded of the true velocity V and the Earth's velocity V_E. It is represented in Fig. 20, by RE.

V_E is the velocity of the Earth, and is not quite constant. If its mean value be taken as unity, then in equation (30) the constant k^2 is also unity, so that

$$V_E^2 = 2/R - 1 \quad \ldots\ldots\ldots\ldots (64)$$

where R is the Sun's radius vector, given in the *Nautical Almanac* for each day of the year.

V_2 is the actual velocity with which the meteor enters the Earth's atmosphere. It is greater than V_1 because the Earth attracts the meteoric particle and forces it to move in a hyperbolic orbit with reference to the Earth's centre. The velocity determined by any experimental method (due allowance being made for retardation, if this should be suspected) is always V_2.

In much of the earlier work the value of V_2 was unknown, and in order to compute an orbit it was always necessary to make assumptions. It is the ability of modern methods to determine V_2 that has made possible the recent advances in our knowledge of meteor streams. Now the attraction

of the Earth for the particles not only augments the velocity from V_1 to V_2 but at the same time changes the direction in which the meteor is moving. The circumstances are illustrated in Fig. 22. The meteor M, at

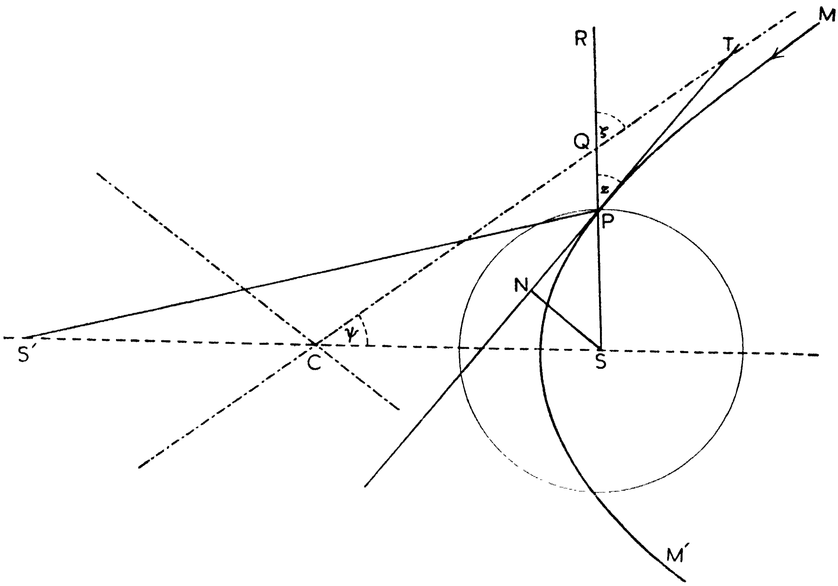


FIG. 22.—Illustrating zenith attraction.

some distance from the Earth, may be considered to be moving along the straight line MC, but as it approaches the Earth more closely, it is forced into the hyperbolic orbit MPM′. This hyperbola will have foci at S (the Earth's centre) and at S′, and if the centre of the hyperbola is at C, then SC = CS′ = a. Also, by the properties of the hyperbola, PS′ = $r + 2a$ where r is the radius of the Earth. An observer at P will measure the zenith distance of the radiant as z (the angle TPR, where TP is the tangent to the hyperbola at the point P) ; whereas the correct zenith distance is ζ (angle TQR, where TQ is an asymptote to the hyperbola). The asymptote TC meets the major axis at an angle ψ such that sec $\psi = e$, the eccentricity of the hyperbola. It is then seen that the radiant is always displaced towards the observer's zenith, and for this reason the displacement is known as *zenith attraction.* Its value may be deduced as follows :

The velocity of the particle relative to the Earth at the point P is given by

$$V_2^2 = \mu\left(\frac{2}{r} + \frac{1}{a}\right) \qquad \text{.............(65)}$$

where μ is now used to represent the acceleration at unit distance from the Earth. The undisturbed velocity V_1 which the meteor would possess outside the range of the Earth's attraction is obtained by making r infinite in this formula, so that

$$V_1^2 = \mu/a \qquad \ldots\ldots(66)$$

from which

$$V_2^2 = V_1^2 + 2\mu/r \qquad \ldots\ldots(67)$$

and

$$V_1^2(2a + r) = V_2^2\, r \qquad \ldots\ldots(68)$$

Now the projections of the sides of a triangle on any given line sum to zero. The sides of the triangle SPS′ may thus be projected on to the asymptote, this line being chosen because the projection of SS′ on the asymptote is simply $2a$, a result which follows from the proposition that the feet of the perpendiculars from the foci on to the asymptotes lie on the auxiliary circle at the ends of a diameter. We thus have (Fig. 22).

$$(r + 2a)\cos(2z - \zeta) - r\cos\zeta = 2a \qquad \ldots\ldots(69)$$

Adding r to each side of the equation and rearranging,

$$r(1 - \cos\zeta) = (r + 2a)[1 - \cos(2z - \zeta)] \qquad \ldots\ldots(70)$$

or, using (68),

$$V_1^2(1 - \cos\zeta) = V_2^2[1 - \cos(2z - \zeta)] \qquad \ldots\ldots(71)$$

Putting $\Delta z = \zeta - z$ so that $2z - \zeta = z - \Delta z$ and $\zeta = z + \Delta z$,

$$V_1 \sin \tfrac{1}{2}(z + \Delta z) = V_2 \sin \tfrac{1}{2}(z - \Delta z) \qquad \ldots\ldots(72)$$

from which

$$\tan \tfrac{1}{2}\Delta z = \frac{V_2 - V_1}{V_2 + V_1} \tan \tfrac{1}{2}z \qquad \ldots\ldots(73)$$

This equation enables the observed radiant to be corrected for zenithal attraction, the correction Δz being added to the observed zenith distance. The actual hyperbolic orbit in which the meteor travels with respect to the earth may be visualised if V_1 and V_2 are known. Thus if d is the length of the perpendicular SN from S upon the tangent,

$$h = dV_2 = rV_2 \sin z.$$

Since

$$h^2 = \mu p$$

and

$$p = a(e^2 - 1) = a\tan^2\psi$$

it follows that

$$h^2 = \mu\, a\tan^2\psi = a^2 V_1^2 \tan^2\psi \qquad \ldots\ldots(74)$$

and hence

$$\tan\psi = rV_2 \sin z/aV_1 \qquad \ldots\ldots(75)$$

This result gives the value of ψ and therefore of the eccentricity of the hyperbola, and the angle between the asymptotes. The semi-major axis is given by

$$a = \mu/V_1^2$$

while the semi-minor axis is given by the product of the two expressions :

$$b = a\tan\psi = rV_2 \sin z/V_1$$

It is now possible to give numerical values to these expressions. The

value of μ to be used may be calculated from

$$\mu = k^2 m$$

in which m is now the mass of the Earth. If this is taken as 0·00000303 of the Sun's mass, and the unit of distance is taken to be that from the centre of the earth to the point of appearance of the meteor—say 6450 km., or 0·0000431 astronomical units, then

$$\frac{2\mu}{r} = \frac{2k^2 m}{r} = \frac{2 \times (29 \cdot 80)^2 \times 0 \cdot 00000303}{0 \cdot 0000431} = 124 \cdot 9$$

so that

$$V_2^2 = V_1^2 + 124 \cdot 9 \qquad \text{.............................}(76)$$

This equation enables the correction given by (73) to be applied once V_1 or V_2 is determined. It will be noticed that the correction for zenith attraction is a maximum when the radiant lies on the horizon, and is near the antapex. In these circumstances, the value of the correction may approach 17°. For approximate working, where the assumption of parabolic motion is sufficient, a set of graphs will enable the correction to be determined very rapidly (Hardcastle, 1910 ; Davidson, 1914) ; in other circumstances, the correction must be computed for each case, or the convenient tables compiled by Nielsen (1938) may be used.

The sphere of action of the Earth

It is a matter of some interest to consider the actual behaviour of a meteoric particle in the neighbourhood of the Earth, and especially to determine the conditions of velocity and distance which may enable a body to miss the Earth completely. The limit is clearly set by a grazing impact, in which case $z = 90°$, and if the radius of the Earth be taken as the unit of distance, $\mu = 62 \cdot 4$, $b = V_2/V_1$, $a = 62 \cdot 4/V_1^2$ and the following table results :

TABLE 18

Action of the Earth on Meteor Streams.

V_1	V_2	b	a	ψ
km./sec.	km./sec.	radii	radii	°
1	11·22	11·22	62·4	10·2
2	11·35	5·68	15·6	20·0
4	11·87	2·97	3·90	37·3
6	12·68	2·11	1·73	50·6
8	13·74	1·72	0·975	60·4
10	15·0	1·499	0·624	67·4
20	22·9	1·145	0·156	
30	32·0	1·067	0·0693	86·3
40	41·5	1·038	0·0390	
50	51·2	1·025	0·0250	88·6
60	61·0	1·017	0·0173	
70	70·9	1·013	0·0127	89·3

The distance of the original path of the meteor from the earth is given by b, which is the distance from S to the asymptote, and it is seen at once that these distances are always very small. In the case of parabolic motion, the relative velocity V_1 lies between $\sqrt{2} - 1$ and $\sqrt{2} + 1$ in terms of the Earth's velocity (i.e., between 12 and 71 km./sec.) so that the particle must approach the Earth within 1·37 radii of the Earth's centre, or less than 2,500 km. from its surface.

The lower limit of speed is difficult to estimate. Theoretically there is no reason why the Earth should not overtake a particle which is at the aphelion of a much smaller orbit, and in such a case the relative velocity of the particle would be very small, while if the meteoric velocity were less than that of the earth, the meteor would appear to be coming from the apex with very small velocity. Such a case has never been accurately recorded, although reports of remarkably slow meteors have been given in the past (Davidson, Cook, 1913). In general, however, it may be concluded that the Earth's sphere of influence on meteoric particles is limited to a radius of a few thousand kilometres, and only under exceptional circumstances can this be extended to hundreds of thousands of kilometres. In any event, the distance is less than 0·005 A.U., and the amount of matter removed by the Earth at each passage through a stream is negligible in comparison with the whole system.

Diurnal aberration

The second correction to be applied to the apparent radiant is that for aberration due to the rotation of the Earth. If the meteor be imagined to be approaching the Earth with velocity V_2, then the position of the radiant will suffer an aberrational displacement K, given by

$$K = v/V_2$$

where v is the velocity of the observer due to the Earth's rotation. This may be taken as 0·4639 km./sec. at the equator, or 0·4639 cos ϕ km./sec. in latitude ϕ. The value of K thus becomes

$$K = \frac{0{\cdot}4639 \cos\phi \times 57{\cdot}296}{V_2} = \frac{26{\cdot}58 \cos\phi}{V_2} \text{ degrees,}$$

and the displacement in R.A. and dec. due to diurnal aberration is

$$\Delta\alpha = -\frac{26{\cdot}58}{V_2}\cos\phi\cos h\sec\delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

$$\Delta\delta = -\frac{26{\cdot}58}{V_2}\cos\phi\sin h\sin\delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (78)$$

where δ is the declination of the radiant, and h its hour angle (h = sidereal time minus R.A.). The correction is only appreciable in the region of the poles, and then only if V_2 is small.

The true radiant

The apparent radiant having been corrected by the application of zenith attraction and diurnal aberration, the corrected radiant may now be used to obtain the true radiant, using the relations between the sides and angles of the parallelogram of velocities shown in Fig. 20. The *elongations* of the apparent and true radiants from the Apex, i.e., the angles REA and TEA respectively, will be referred to as ε and ε'. (No confusion is likely to arise with the same symbol used in previous work for the obliquity of the ecliptic.) Then the velocities and elongations are connected by the equations :

$$\frac{V}{\sin \varepsilon} = \frac{V_1}{\sin \varepsilon'} = \frac{V_E}{\sin (\varepsilon' - \varepsilon)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (79)$$

and these equations may be used in various ways to compute the quantities required. The following cases may arise :

(1) Given V_2 and ε—this is the ordinary observational case. Compute V_1 from (76), and rewrite equation (79) in the form

$$\cot \varepsilon' = \cot \varepsilon - \frac{V_E}{V_1} \operatorname{cosec} \varepsilon$$

$$\text{whence } V = V_1 \frac{\sin \varepsilon}{\sin \varepsilon'}$$

(2) Given ε and the period or size of the orbit. Use (28) to obtain a from the period, and then (30) to obtain V.

Compute ε' from

$$\sin (\varepsilon' - \varepsilon) = \frac{V_E}{V} \sin \varepsilon$$

$$\text{whence } V_1 = V \frac{\sin \varepsilon'}{\sin \varepsilon} \text{ and } V_2 \text{ follows from (76).}$$

If nothing is known as to the orbit, it is usual to assume parabolic motion, in which case V is taken as $\sqrt{2/R}$ so that

$$\frac{V_E}{V} = \sqrt{1 - R/2}$$

(3) In the reverse problem of computing a radiant from a given orbit, it is necessary to compute V_2 and ε from a knowledge of V and ε'. In this case,

$$\cot \varepsilon = \cot \varepsilon' + \frac{V_E}{V} \operatorname{cosec} \varepsilon'$$

$$\text{and } V_1 = V \frac{\sin \varepsilon'}{\sin \varepsilon}$$

The elongation of the radiant

The position on the celestial sphere of the radiants R and T and their relation to the Sun S and the Apex A are shown in Fig. 23. The position of the true radiant is obtained as follows :

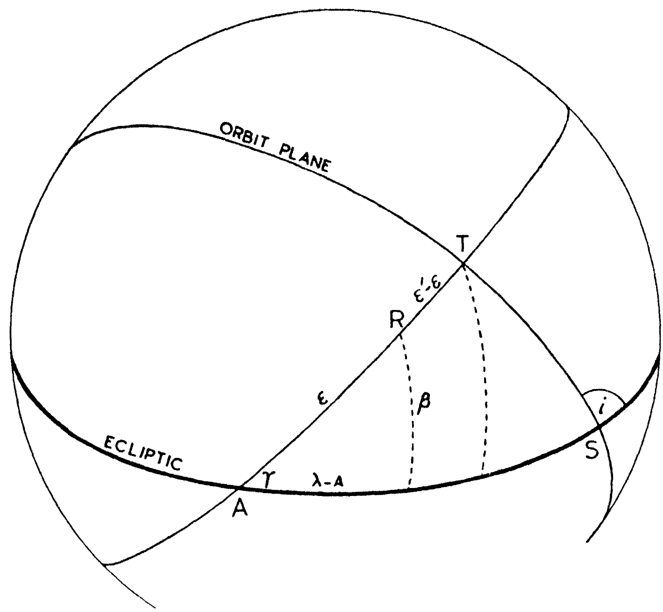


FIG. 23.—The true radiant and the plane of the orbit.

The R.A. and dec. of the corrected radiant are first converted to longitude and latitude (λ, β) (Appendix 2), and denoting the angle TAS by γ,

$$\begin{aligned} \cos\beta\cos(\lambda - A) &= \cos\varepsilon \\ \cos\beta\sin(\lambda - A) &= \sin\varepsilon\cos\gamma \\ \sin\beta &= \sin\varepsilon\sin\gamma \qquad \ldots\ldots\ldots\ldots\ldots\ldots(80) \end{aligned}$$

from which ε and γ may be obtained. Knowing ε, the value of ε' is determined from (79) and equations (80) may then be used again, if the longitude and latitude of the true radiant are required. Normally, however, these quantities are not necessary, and it is convenient to pass straight on to the calculation of the orbit.

Since T, S and the observer are in the plane of the orbit (Fig. 23) the great circle TS is the projection of this plane on the celestial sphere, and the angle TSA is $180° - i$, so that the elements of the orbit may be determined by a solution of the spherical triangle TSA (Bauschinger, 1928 ; Olivier, 1925 ; Davidson, 1934). This method involves a number of inconvenient rules which may be avoided by the use of direction cosines

throughout. The following method is similar to that of Laplace for cometary orbits. Representing the components of the meteor's velocity by x', y', z' (the Earth's mean velocity being the unit), equation (30) may be written

$$V^2 = 2/r - 1/a = (x')^2 + (y')^2 + (z')^2 \qquad \ldots\ldots\ldots(81)$$

and the radius vector r is obtained from

$$r^2 = x^2 + y^2 + z^2 \qquad \ldots\ldots\ldots(82)$$

which on differentiation gives

$$rr' = xx' + yy' + zz' \qquad \ldots\ldots\ldots(83)$$

Now equations (12) and (13) may be written in the form

$$\begin{aligned} e \cos v &= p/r - 1 \\ e \sin v &= r'\sqrt{p} \end{aligned} \qquad \ldots\ldots\ldots(84)$$

from which, on squaring and adding, and using $p/a = 1 - e^2$

$$p = V^2r^2 - (rr')^2 \qquad \ldots\ldots\ldots(85)$$

The projection of the areal velocities on the planes of reference are given by (52), and since it is known that the earth is at one of the nodes of the meteor orbit, the remaining elements can be obtained from these equations.

Now the co-ordinates x, y, z are those of the Earth at the given time, while the velocity V is either determined experimentally, or, more commonly, is based on an assumption of the value of a and the use of (30). If nothing is known as to the orbit, then it is usual to assume parabolic motion, in which case a is infinite. The position of the radiant is corrected as explained in the last chapter, and the position of the true radiant is computed in terms of longitude and latitude, λ', β'. The positions and velocities will be referred for convenience to the plane of the ecliptic, but the X-axis will be assumed to be directed towards the apex. On this system the direction cosines F, G, H of the corrected true radiant will be given by

$$\begin{aligned} F &= \cos\beta' \cos(\lambda' - A) &&= \cos\varepsilon' \\ G &= \cos\beta' \sin(\lambda' - A) &&= \sin\varepsilon' \cos\gamma \\ H &= \sin\beta' &&= \sin\varepsilon' \sin\gamma \end{aligned} \qquad \ldots\ldots\ldots(86)$$

and as the meteor is moving from the radiant, the components of its velocity will be

$$x' = -VF, \quad y' = -VG, \quad z' = -VH \qquad \ldots\ldots(87)$$

The heliocentric co-ordinates of the earth, referred to the same system will be

$$x = -R\sin D, \quad y = -R\cos D, \quad z = 0 \qquad \ldots\ldots\ldots(88)$$

since the apex is in advance of the Earth by $90^\circ - D$, and the Earth's radius vector is R, as given in the *Nautical Almanac*. Equations (52)

must now be referred to the same system of axes, and projecting the areal velocities on to the XY, YZ and ZX planes,

$$\begin{aligned} \sqrt{p}.\cos i &= xy' - yx' &&= RV(G\sin D - F\cos D) \\ \sqrt{p}.\sin i.\cos D &= yz' - zy' &&= RV.\,H\cos D \\ -\sqrt{p}.\sin i.\sin D &= zx' - xz' &&= RV.\,H\sin D \quad \ldots.(89) \end{aligned}$$

Since D is always less than 1°, its cosine may be written as unity with almost four-figure accuracy, and the various equations may be further simplified, by making the substitutions

$$\begin{aligned} B &= G\sin D - F \\ C &= G + F\sin D \\ S^2 &= 1 - C^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(90) \end{aligned}$$

which lead to the following simple expressions :

$$\begin{aligned} \sqrt{p} &= VRS \text{ (compare equation (34))} \\ e\sin v &= VC\sqrt{p} \\ e\cos v &= p/r - 1 \\ S\sin i &= H \\ S\cos i &= B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(91) \end{aligned}$$

The value of the true anomaly v leads to the value of ω, while the time of observation gives the value of ☊. When the radiant is *north* of the ecliptic, the Earth must be at a descending node, so that ☊ $= L$, and since both ω and v are measured in the direction of motion of the meteor, $\omega + v = 180°$. Conversely, if the radiant is *south* of the ecliptic, ☊ $= L \pm 180°$ and $\omega + v = 360°$. The actual behaviour of the meteor may be visualised by consideration of the value of the true anomaly v. Thus if the meteor is approaching perihelion then $v > 180°$, and r is decreasing, so that C and $\sin v$ are negative. If the meteor has passed perihelion, then $v < 180°$, r is increasing and C and $\sin v$ are positive. If the motion is retrograde, these rules still apply, but in order to ensure that $\sin i$ is always positive, it is necessary to make H positive, so that the angles β', γ, ε and ε' must always be regarded as less than 180°. Further details, and examples of the calculation, are given in Appendix 3.

Cometary meteor orbits

The reverse problem, that of determining the radiant of a stream associated with a given comet, may be solved by using the same formulae. As may be seen from the distances given in Table 17 on p. 78, the approach of a comet to the Earth's orbit must necessarily be very close, but in order to cover all possible conditions, a maximum distance of 0·1 unit may be assumed, although even this may give rise to considerable errors in the computation. The results of such a calculation must for this reason be regarded as approximate only.

Any comet which can make a sufficiently close approach must have an orbit in which the value of q is less than 1·1, and in the circumstances of that approach four different cases may arise :

(1) In the majority of orbits, the nearest approach will take place at one of the nodes of the comet's orbit. Compute

$$r = \frac{p}{1 \pm e \cos \omega} \quad \text{where} \quad p = a(1 - e^2).$$

If the use of the *positive* sign gives a value of r between 0·9 and 1·1, the Earth will intersect the stream at ☊ and the radiant will be *south* of the ecliptic ; also $L =$ ☊ $\pm 180°$ and $v = 360° - \omega$. If the *negative* sign applies, the radiant is *north* of the ecliptic, $L =$ ☊ and $v = 180° - \omega$. Knowing L, the date and value of R may be obtained from the *Nautical Almanac*, and hence the distance C − E $= r - R$.

(2) In some cases the nearest approach will be at some distance from the node (Fig. 24). This will occur when the inclination is small (less than

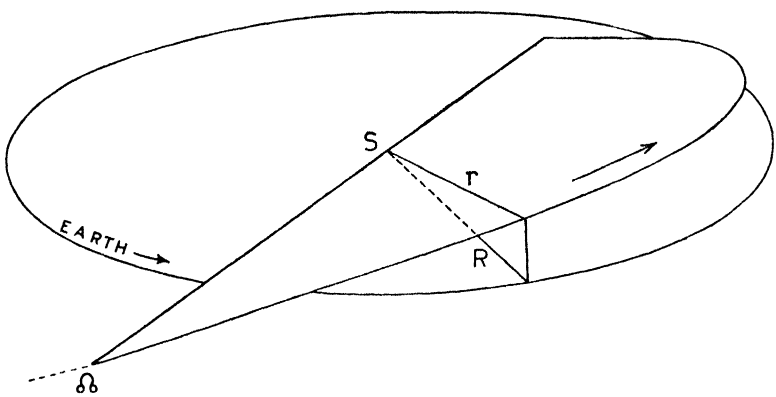


FIG. 24.—Closest approach of a meteor stream.

6°, if the distance 0·1 is taken as a criterion) and $q = a(1 - e)$ must be less than unity. The point of nearest approach may be taken with sufficient accuracy to be that at which $r = R$. As a first approximation take $R = 1$, so that

$$\cos v = \frac{p - 1}{e}$$

Of the two possible values of v choose that which brings the meteor within 0·1 unit of the Earth's orbit, i.e., the value of v is such that

$$\sin (v + \omega) \sin i < 0{\cdot}1$$

With this value of v, the position of the Earth is ☊ $+ v + \omega$ and that of the Sun ☊ $+ v + \omega \pm 180°$. Knowing L, the date and value of R may be

obtained, and the value of v improved, if thought necessary, by using the more accurate formula

$$\cos v = \frac{p - R}{Re}$$

The distance is given by

$$C - E = R \sin (v + \omega) \sin i \quad \text{with sufficient accuracy.}$$

(3) In a few cases a comet, having q slightly greater than unity, and with a very small inclination, makes its closest approach at perihelion. Under these circumstances, $v = 0$, and the remaining quantities may be obtained as under (2).

(4) It is also possible for a meteor stream to move in an orbit which is sufficiently close to that of the Earth to give radiants over a wide range of values of v. In this case also the inclination must be very small, and a series of radiants must be computed for various assumed values of v.

If the orbit being investigated is a parabola, some simplification is possible. Thus

$$r = q/\sin^2 \tfrac{1}{2}\omega \quad \text{or} \quad r = q/\cos^2 \tfrac{1}{2}\omega$$

will decide the distance, the first formula corresponding to a northern radiant, and the second to a southern radiant.

In any case, having found L from ☊, obtain D, $\sin D$ and V_E as before. Compute

$$V^2 = \frac{2}{r} - \frac{1}{a}$$

$$S = \frac{\sqrt{p}}{Vr} \qquad C = \frac{e \sin v}{\sqrt{p}\, V} \qquad \textit{Check}: S^2 + C^2 = 1$$

Then

$$\begin{aligned}
F &= S \sin i &&= \sin \varepsilon'. \sin \gamma \text{ (always positive)}\\
B &= S \cos i &&\\
G &= C + B \sin D &&= \sin \varepsilon' \cos \gamma\\
H &= C \sin D - B &&= \cos \varepsilon'\\
\textit{Check}&: F^2 + G^2 + H^2 &&= 1
\end{aligned}$$

and from these obtain ε' and $\sin \gamma$ and $\cos \gamma$. The value of ε follows from

$$\cot \varepsilon = \cot \varepsilon' + \frac{V_E}{V} \operatorname{cosec} \varepsilon'$$

and λ and β are determined from

$$\begin{aligned}
\cos \beta \cos (\lambda - A) &= \cos \varepsilon\\
\cos \beta \sin (\lambda - A) &= \sin \varepsilon \cos \gamma\\
\sin \beta &= \sin \varepsilon \sin \gamma \text{ (always positive).}
\end{aligned}$$

β will be taken as north or south according to the rules given for the preliminary step.

It will be noticed that this method makes assumptions that are inherent in the problem, and cannot be avoided. These are that the meteor, although not travelling precisely in the comet's orbit, has the same direction of motion, and the same velocity as the comet. In other words, the difference between r and R is ignored (the latter is not used in the calculation of the radiant) and for this reason must not be too large. Previous lists of possible close approaches have been given by A. Herschel (1875) and Davidson (1920), but in these cases a limit of 0·25 unit has been allowed. If the lower limit is used, the number of possible cases is quite small, and those which can arise from the elliptical orbits of later date than 1700 in the present comet list are given in Table 15.

TABLE 19

Cometary Meteor Radiants.

Date	Radiant	Comet	Period (years)	Closest at	C − E	
	° °			°		
Jan. 9	330−39	1819 IV	5·1	☊ +29	+0·077	
Feb. 11	350−10	1743 I	5·4	☊ +52	+0·026	
Mar. 30	308−61	Grigg-Mellish	164	☊	−0·002	
Apr. 21	271+34	1861 I	417	☋	−0·002	Lyrids
Apr. 26	109−37	Grigg-Skjellerup	5·0	☊	−0·098	
May 6	336− 1	Halley	76·3	☋ −11	−0·064	Aquarids
June 8	218+45	1930 VI	5·4	☋	+0·006	
June 30	204+56	Pons-Winnecke	6·1	☋	+0·028	Draconids
July 5	272−21	Lexell 1770 I	5·6	☋ −32	+0·015	
Aug. 11	45+58	1862 III	120	☋	+0·010	Perseids
Sept. 29	278−37	Finlay	6·7	(peri.)	+0·069	
Oct. 10	262+54	Giacobini-Zinner	6·6	☋	+0·004	Giacobinids
Nov. 11	147+24	1866 I	33·2	☋ − 5	−0·024	Leonids
Nov. 14	21+ 3	1743 I	5·4	☊ −39	−0·021	
Nov. 28	23+42	Biela	6·6	☋ − 2	+0·008	Andromedids
Dec. 5	256−25	Lexell 1770 I	5·6	☊ −62	−0·025	
Dec. 15	103+ 9	1917 I	145	☊ − 6	−0·061	
Dec. 17	277−35	Denning 1881 V	8·5	☊ +18	+0·037	
Dec. 22	219+74	Tuttle 1926 IV	13·5	☋	+0·100	

Seven of these theoretical radiants agree with those of well-known showers, and these have been referred to in connection with Table 17. Of the others, W. F. Denning suggested from time to time a number of cases of accordance between comets and the radiants of minor showers, and in 1923 published a list of 28 such accordances. Apart from the seven showers mentioned, none of these cometary meteor radiants had received independent confirmation until Bečvář announced his discovery of the December Ursids on 1945 December 22 with a radiant at 233° + 83°. The 1946 observations gave a radiant at 203° + 75°, and the presence of this stream was later confirmed by radar methods. The connection of the

December Ursids with Tuttle's comet has been shown by Hawkins and Almond (1950).

The remaining comets of long period, or having parabolic orbits, make sufficiently close approaches to the Earth in 68 cases, but although some of these appear in Denning's list, the agreement must be considered as purely accidental. There would appear to be no reason at all to associate recurrent streams with comets having parabolic orbits.

BIBLIOGRAPHY

REFERENCES

BAUSCHINGER, J., 1928, *Die Bahnbestimmung der Himmelskörper*, 2nd edition (Leipzig: Engelmann), Chap. 35.

COOK, A. G., 1913, *Brit. Astr. Assocn. J.*, **24**, 168.

DAVIDSON, M., 1913, *Brit. Astr. Assocn J.*, **24**, 101.

DAVIDSON, M., 1914, *Brit. Astr. Assocn. J.*, **24**, 307.

DAVIDSON, M., 1920, *Mon. Not. Roy. Astr. Soc.*, **80**, 739.

DAVIDSON, M., 1934, *Brit. Astr. Assocn. J.*, **44**, 116 and 146.

DENNING, W.F. 1923, Hutchinson's *Splendour of the Heavens*, Chap. x, p. 440.

HARDCASTLE, J. A., 1910, *Brit. Astr. Assocn. J.*, **21**, 154.

HAWKINS, G. S., and ALMOND, M., 1950, *Brit. Astr. Assocn. J.*, **60**, 251.

HERSCHEL, A., 1875, Report on *Meteors* to the British Association.

NIELSEN, A.V., 1938, *Tables for determination of perturbations of meteors by the Earth, Medd. Ole Rømer Obs.*, Nr. 12 (in English).

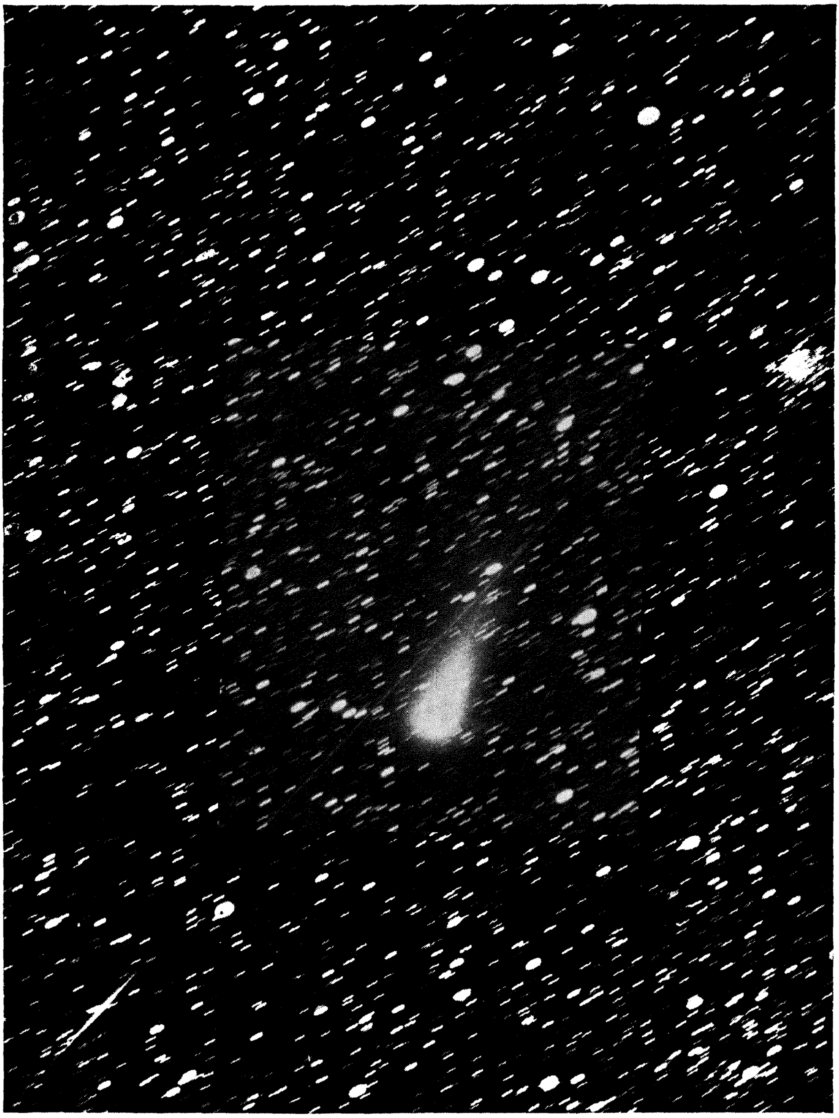
OLIVIER, C. P., 1925, *Meteors* (Baltimore : Williams and Wilkins) Chap. XV.

CHAPTER VII

Meteor Velocities

The belief in the presence of hyperbolic velocities among sporadic meteors has not arisen as the result of any direct observations, and, in fact, the observational work of the past century would suggest that no such abnormal velocities exist. Duplicate observations of meteors by reliable observers (whose ability to make accurate records of such transient phenomena is one of the outstanding characteristics of amateur astronomy) enable the true path of the meteor in the atmosphere to be computed and the velocities estimated. Although these estimated velocities can hardly be regarded as accurate, there has never been the slightest indication of any large numbers of meteors having speeds in excess of the parabolic limit. Apart from the results of such calculations, skilled observers like Denning and Prentice would hardly fail to notice any marked differences between the apparent speeds of sporadic meteors and those of the annual showers with which they were so familiar. Such differences would be apparent, for example, in the angular velocities of the two types of meteor, yet although the slightest abnormality was faithfully recorded, no such differences have ever been noticed that were not capable of explanation in terms of parallax and perspective.

It is not surprising, therefore, that the skilled observer should give little credence to the deductions as to the velocities of meteors that were made on a basis of analysis of the hourly rates. The early work of Schiaparelli and von Niessl was extended by Hoffmeister to obtain a mean value for the velocity of meteors on the basis of the same assumption of a uniform distribution of true radiants. His meteor counts in different latitudes led to a mean velocity of 60 to 80 km./sec., or 2 to 3 times the Earth's mean orbital velocity. In his later work, Hoffmeister calls these values " apparent effective velocities," thus implying that the fundamental assumption of a uniform distribution of original directions is not, in actual fact, fulfilled. In much of his work, Hoffmeister was able to show that the variation in hourly rate is not a simple function of heliocentric velocity alone, but depends also on the irregularities of distribution of the radiants. Nevertheless, he inferred from the daily and seasonal variation in the hourly rate that there exist two great preferential directions of motion of meteors (the so-called interstellar streams), one from the constellation of Scorpio, and the other from Taurus. The dust particles of such streams



COMET PELTIER, 1936 II
and a meteor trail. Exposure 1 h. 30 m.

(*Quénisset, Juvisy*)

Facing p. 94

would necessarily describe hyperbolic orbits about the Sun, and the resulting meteors would have hyperbolic velocities.

An apparently strong confirmation of these high velocities was given by the results of the Harvard-Cornell Meteor Expedition to Arizona in 1931–3, a great undertaking which was designed to give a survey of the universe of meteors. The campaign began on a high note :

" The nature of the interstellar and intergalactic media through which radiation, stars, clusters and galaxies move is found to be of so much significance in our understanding of galactic distances and structure that fundamental research on the contents of space has become necessary. One aspect of the problems are the meteors. Investigation of these multitudinous small bodies directly bears not only on knowledge of their own physical nature and their place in the cosmic structure, but as well on the question of the content of interstellar space ; and indirectly such investigations may contribute to the solution of the general problem of ' planetesimals ' in the origin of the solar system, and of the structure of the upper terrestrial atmosphere." (Shapley, Öpik and Boothroyd, 1932.) It is unfortunate that such an enterprise should have given results which are at complete variance with those of any other method of measurement, and which remain an unexplained anomaly in the field of meteor research.

In the Arizona expedition the observers carried out a watch on selected areas of the sky, viewed through a reticle which enabled the apparent flight-direction to be determined. The duration of flight could be estimated in the usual way, but a considerable proportion of the meteors were sufficiently well placed to enable the duration to be measured by means of an ingenious " rocking-mirror " device. In this instrument a plane mirror executes an oscillatory motion in which the normal to the mirror describes a conical surface ten times a second. The image of a meteor trail thus becomes a cycloid which can still be traced on a star-map, and the angular velocity of the meteor can be determined either from the number of loops in the trail or from the shape of the loops.

Many thousands of observations were made, and the work was continued by Öpik in Estonia from 1934 to 1941. The output was truly enormous, and the variety of topics that were discussed covered every possible aspect of the subject. It must be emphasised, however, that the stated object of Öpik and his collaborators was to obtain data of statistical value, and the methods used were not such as to give direct measurements. (Prentice, 1932.) Thus it was not possible to obtain individual values for radiants or heights, and it is an essential feature of this work that it was necessary to assume certain mean centres of radiation ; while the linear velocities were computed from the angular velocities and assumed

mean heights. The latter were based on empirical formulae involving the angular velocities themselves and the luminosities.

The resulting analysis showed that there were marked differences between shower and non-shower meteors, the latter being far more numerous and having a considerable excess of hyperbolic velocities. Nearly 70 per cent of the sporadic meteors were considered to have velocities greater than the parabolic limit.

The fact that the results of the Arizona Meteor Expedition were obtained as the result of a statistical analysis of data built on a foundation of assumptions has been strongly criticised (Porter, 1944). Thus the very quantities which are used as the basis of the assumptions (i.e., heights, luminosities) are known to be strongly correlated with the velocities which are being investigated. They are therefore themselves capable of being used in a statistical comparison of groups of meteors. Porter has shown that meteor observations reduced by a common method so that all are equally subject to the same kinds of error, can be divided into groups of comparable elongation (from the apex) and brightness, and then show remarkable similarities. Thus a collection of 778 duplicate observations made by experienced observers in this country gave heights and path-lengths of considerable accuracy. The velocities were not so well determined, since the visual observer is not able to estimate the duration with sufficient accuracy, but a comparison of the velocities of shower and non-shower meteors can still be made. For 298 shower meteors, the mean velocity was 0·67 of the parabolic velocity, and exactly the same figure was obtained as the mean velocity for the 480 non-shower meteors.

Such values are easily criticised on the grounds of lack of accuracy, or because the sampling, particularly of the shower meteors, is necessarily stratified. The comparison of the more accurate heights can, however, be made under more nearly equivalent conditions, and such a comparison is made in Table 20, between certain major—and minor—shower meteors and corresponding groups of sporadic meteors. Group A contains those meteors having the same range of dates, of elongation ε and of magnitude M, as the shower meteors ; while Group B has the same range in ε and M but covers *all other dates* in the year. It is clear that the mean heights are significantly the same in corresponding groups. No particular care was taken over the selection, all those meteors coming within the required ranges being used. Had the ranges been further restricted, so as to give the same *means* in ε and M, or similar restrictions applied in other measurements, it is to be anticipated that even closer agreement would be found. There is thus no doubt that the heights of appearance and disappearance of a meteor are determined mainly by the

values of ε and M, both of which are known to be correlated with velocity. The actual laws governing this correlation are still in dispute, and a great deal of work remains to be done in this connection.

TABLE 20

Mean heights of groups of meteors.

		Shower	Sporadic	
			Group A	Group B
Leonids $\varepsilon = 10^\circ{\cdot}5$	H_1 H_2 n	125·6 ± 2·34 94·1 ± 2·00 27	— — —	121·2 ± 4·52 101·3 ± 3·17 21
Orionids $\varepsilon = 24^\circ{\cdot}1$	H_1 H_2 n	119·7 ± 2·14 100·9 ± 1·79 35	117·0 ± 4·06 99·5 ± 2·77 14	114·4 ± 1·90 101·1 ± 1·65 53
Perseids $\varepsilon = 39^\circ{\cdot}2$	H_1 H_2 n	114·9 ± 1·32 94·7 ± 0·98 152	108·8 ± 2·15 93·2 ± 2·73 26	112·0 ± 2·56 94·4 ± 2·20 29
δ-Aquarids $\varepsilon = 69^\circ{\cdot}4$	H_1 H_2 n	101·1 ± 4·43 89·0 ± 4·39 15	101·9 ± 2·99 84·9 ± 1·96 16	100·3 ± 2·23 83·8 ± 2·49 36
Taurids $\varepsilon = 81^\circ{\cdot}4$	H_1 H_2 n	101·6 ± 2·24 77·7 ± 1·97 28	90·9 ± 4·43 78·8 ± 3·49 14	98·8 ± 2·94 77·9 ± 1·94 62
α-Capricornids $\varepsilon = 93^\circ{\cdot}0$	H_1 H_2 n	95·5 ± 4·25 85·3 ± 3·42 14	100·9 ± 3·24 82·7 ± 2·24 13	93·3 ± 2·09 80·5 ± 2·48 37

H_1 = height in kms. of beginning of observed path.
H_2 = height of end of path. n = number of values.

The widely divergent results of the two forms of indirect analysis can be critically examined only in the light of direct measurements, which are now becoming available in increasing numbers in the results of photographic and radar research. The work at Harvard continues, and the early photographic results of Elkin and of Millman and Hoffleit (1937), which showed that high accuracy could be attained, have been extended by Whipple to a comprehensive survey of photographically-determined meteor velocities. The results showed, even in 1938, that meteors could travel in small orbits like those of a minor planet, but perhaps the outstanding feature of Whipple's work has been the accurate measurement of the geocentric velocities of meteors of the Geminid and Taurid showers (Whipple, 1940, 1947 ; Wright and Whipple, 1950). The figures obtained

led to the discovery of the remarkably small orbits in which these streams travel. The Taurids give a display each year from about October 26 to November 22 (a prolonged period which had previously been explained as due to hyperbolic orbits), and duplicate photographs of Taurid meteors provided accurate values of the velocities from which the orbit could be computed. The mean of five well-determined orbits gave the following figures :

	°		
ω	113·5	a	2·350
☊	43·6	e	0·848
i	5·3		
q	0·355	P	3·604 years.

Whipple draws attention to the general similarity between these orbits and that of Encke's comet, the principal difference being in the position of the nodes, causing a divergence of 10°–15° between the planes of the meteor and comet orbits. In Fig. 25 the orbit of Encke's comet is shown,

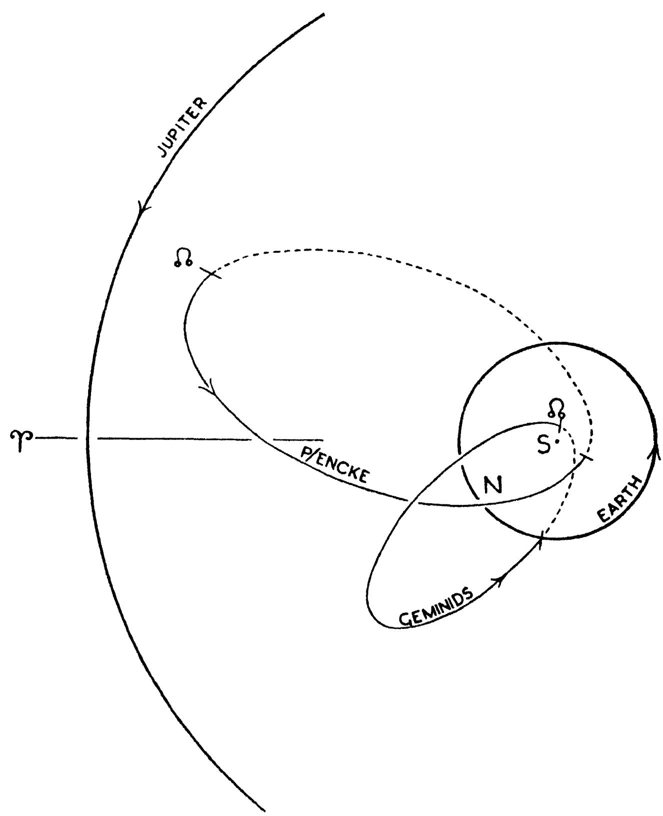


FIG. 25.—The orbits of P/Encke, the Taurids and Geminids.

and on this scale it would be almost exactly similar to that of the Taurids. The ascending node of the meteor stream, however, occurs at the point N. Whipple has substantiated this identification by a process of extrapolation of the perturbations of Encke's comet, making use of formulae similar to (59). This method is necessarily approximate, since any process of extrapolation must always be accepted with considerable reserve ; but it sufficed to show that the Taurids and Encke's comet must have had identical orbits about 13,000 years ago.

In the case of the Geminids, the orbits proved to be smaller than those of any known comet or meteor stream (Fig. 25). The mean figures for five orbits determined in December 1947 were as follows :

	°		
ω	324·3	a	1·396
Ω	260·7	e	0·900
i	23·5		
q	0·139	P	1·65 years

An additional 36 single trails were used to determine the position of the mean radiant and its daily motion, which was shown to be 53′ a day eastward. The surprisingly small values of these meteoric orbits are a notable feature of modern work, and similar results have been obtained by the radar team at Manchester from measurements of the velocity of shower meteors. The determination of size in these small orbits is capable of considerable accuracy. This arises from the fact that an error in a, caused by an error in the measurement of V, increases rapidly with the size of the orbit. The value of a will be determined from (30), which on differentiation gives

$$\Delta a = 2a^2V.\,\Delta V$$

and in a small orbit, both a and V have low values. Thus considerable reliance may be placed on these photographic results, although the number of measures is as yet very few.

The first velocity determinations by radar methods were made by Hey, Parsons and Stewart (1947) during the remarkable Giacobinid shower of 1946. In their apparatus, the echoes were displayed on the cathode ray tube as bright spots (intensity modulation) on a linear range trace. This display was photographed on a film moving at right angles to the trace and several of these records showed a faint fast-moving echo just before the formation of the main echo. This was attributed to the approaching head of the column of ionised gases, and its velocity could be measured from the photographs. The velocities deduced from 22 tracks gave a mean value for the Giacobinids of 22·9 $\pm$ 1·3 km./sec., in good

agreement with the theoretical figure of 23·7 km./sec. The phenomenon is more readily observed at lower frequencies, and numerous measurements have been made in this way by Millman and McKinley (1950) at Ottawa, using a frequency of 30 Mc./sec.

The early results of the radar workers suggested that the radio echo was of a very complex character, and Herlofson (1948) suggested that during the process of formation of the column of ionised gases, the radio waves would be scattered from the growing column, giving rise to a diffraction pattern similar to that of light at a straight-edge. The observation of this pattern was successfully achieved by Davies and Ellyett (1949) and became the basis of an ingenious method of velocity determination. The apparatus employs two cathode ray tubes, the first of which measures the range on a short time base, while the second displays the varying amplitude of the echo at intervals of the order of 1–2 milliseconds, over the first tenth of a second of the life of the echo. The variation of amplitude is periodic, and the distances between successive maxima or minima may be shown to be in the ratios of the zone-widths given by diffraction theory. The time to traverse any zone is readily measured from the number of pulses displayed between maxima, and since the range is given by the first C.R. tube, the velocity may be determined.

The instrument is entirely automatic, being triggered by the echo from the meteor trail. Many of the resulting photographs are wasted, since the apparatus is triggered by unwanted noise, while only a proportion of the meteors shows the typical diffraction pattern. Nevertheless, the method promises to give the large-scale results which are vital to an accurate statistical analysis. It was first used in measuring the velocities of the Geminids in December 1947 and of the Quadrantids in January 1948, and has been in almost continuous operation since that time. The daylight streams were further investigated in 1948 (Aspinall, Clegg and Lovell, 1949) but this time the velocities were measured (Ellyett, 1949). The work was continued in 1949–1950 (Davies and Greenhow, 1951), and the orbits computed from these values by Miss Almond (1951) are also found to be small, and in every way comparable with those found by Whipple.

The radiants found for these summer daytime showers move progressively from Pisces to Taurus (Clegg, Hughes and Lovell, 1947) and the four principal showers give orbits which are shown in Fig. 26. These are based on mean dates for each shower, and as the principal shower, the Arietids, lasts for 18 days, the diagram gives only a slight indication of the complex network of intersecting orbits that must exist in this region of space. The inclinations of many of these orbits are quite small and it is therefore theoretically possible for the streams to give a meteor shower

at the opposite node. In three cases, identification of certain night-time showers with daytime streams has been suggested :

Daytime	Night	Date	Radiant
			° °
β-Taurids		June 30	86 + 19
	Taurids	Nov. 3–10	55 + 20 (mean)
Arietids		June 8	44 + 23
	δ-Aquarids	July 28–30	336 − 11
ζ-Perseids		June 8	62 + 24
	Arietids	Oct. 12	38 + 12

The daytime Taurids are thus also associated with Encke's comet, but otherwise these orbits show little resemblance to those of comets, but seem to be more closely allied to those of the minor planets.

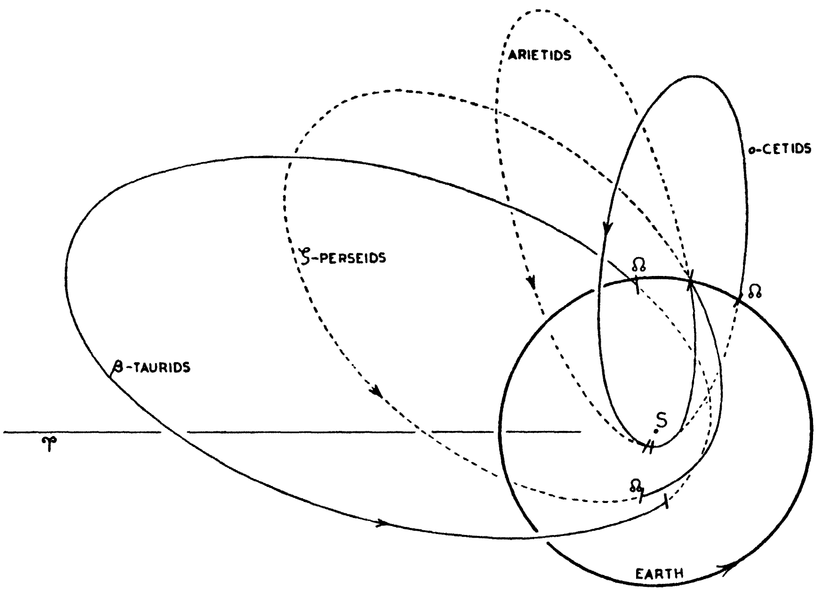


FIG. 26.—Orbits of the daytime streams.

The complexity of the amplitude fluctuations in the reflected signal has been further investigated, and it is concluded that these are due to wind gradients in the upper atmosphere (Greenhow, 1950). The influence of these high winds on meteoric ionisation has been studied in this country (Ellyett, 1950) and also has a considerable bearing on the metero-logical and ballistic research which forms a large part of the Harvard Meteor programme. This utilitarian aspect of meteor research forms,

however, an entirely separate branch of the subject, and the present purely astronomical side of the work is concerned mainly with the question of meteor velocities.

The complete absence of any velocities in excess of the parabolic limit has been a characteristic feature of all radar and photographic work. In an attempt to decide whether hyperbolic velocities exist, Lovell and his colleagues have made a series of experiments on the sporadic meteors designed to measure the velocities of particles travelling from or towards the apex (Almond, Davies and Lovell, 1950, 1951). The first experiments in 1948 used a narrow beam aerial transmitting on 4 m. and directed towards the east, and records were made in the autumn mornings between 5^h and 7^h. During these hours, the apparatus would record meteors that were moving approximately in a great circle plane through the apex, and the 67 velocities recorded gave a distribution which had its peak in the region of 60–65 km./sec. The results are believed to refer to meteors down to the fifth visual magnitude, but they were criticised by Hoffmeister on the grounds that the technique fails to record velocities above the parabolic limit. The mean duration of the echoes decreases with increasing velocity, and there is thus a limit on the measurement of the higher velocities, particularly those over the parabolic limit (72 km./sec. at the apex).

The experiments were therefore repeated a year later, using a wavelength of 8 m., which would raise the instrumental cut-off to about 140 km./sec. The same results were again obtained, and the distribution of 185 velocities showed excellent agreement with the theoretical distribution which would arise for meteors having parabolic velocities and random directions of motion. A more convincing proof was obtained in the spring of 1950 by using the same 8 m. equipment to measure the velocities of meteors moving in a great circle plane through the antapex. This was achieved by a watch from 17^h to 19^h in the spring months, and 86 velocities were obtained ; but there was now a marked discrepancy between observed and theoretical velocity-distributions, the meteors travelling much more slowly than would be anticipated on the basis of random parabolic motion. Two possible explanations, or a combination of these, have been suggested. Either the meteors travel in markedly elliptical orbits, or else their orbits have small inclinations, the radiants being concentrated towards the plane of the ecliptic.

This work was extended to fainter meteors in later experiments on 8 m., using more sensitive equipment. In the autumn of 1950 the apex experiment gave 335 measures of velocity, while fifty-seven velocities were determined in the antapex experiment in the spring of 1951. There was again no evidence of hyperbolic velocities, while the discrepancy between

the theoretical distribution and the measured velocities in the antapex experiment was again confirmed. A more detailed investigation of the magnitude range of these meteors suggests that the 1949–1950 results referred to meteors of magnitudes 4·5 to 6·0, while the more sensitive apparatus of 1950–1951 recorded meteors fainter by 1·5 magnitudes. The experiments are being continued with equipment of even greater sensitivity, but the results to date give no evidence for a significant hyperbolic velocity component such as that found by Öpik and Hoffmeister.

Similar results have been obtained by Millman and McKinley (1950) at Ottawa, using a different technique on 30 Mc. Up to June, 1950, some 11,000 measures of meteor velocities had been made by the Canadian workers, and these have given no evidence of hyperbolic velocities, although measurements of velocity up to 150 km./sec. are possible with this equipment, and reasons are given for considering that it will cover the range of meteors down to the 7th or 8th visual magnitude. On the assumption that the majority of sporadic meteors travel in elliptical orbits of low inclination to the ecliptic, the calculated diurnal distribution of velocities agrees well with the observed distribution.

The conclusion is inevitable that all meteors, like comets, are members of the solar system. A small number of hyperbolic meteors must be expected, since perturbations alone will produce these, exactly as in the case of comets. The velocities so obtained will be only slightly in excess of the parabolic value, and effect will be to cause some small modification to the tail of the distribution curve. The main facts are clear enough, and in the future the new methods of measurement may be expected to give a mass of data, free from ambiguity and assumption, the statistical analysis of which will give an increased knowledge of the structure of the solar system.

BIBLIOGRAPHY

A. BOOKS

HOFFMEISTER, C., 1937, *Die Meteore* (Leipzig : Akad. Verlag.).

HOFFMEISTER, C., 1948, *Die Interstellare System der Kleinkörper* (Verlag Werden und Wirken Weimar) (Part in English).

LOVELL, A. C. B., and CLEGG, J. A., 1952, *Radio Astronomy* (London : Chapman and Hall).

OLIVIER, C. P., 1925, *Meteors* (Baltimore : Williams and Wilkins).

B. REPORTS AND LECTURES

HERLOFSON, N., 1948, *Theory of Meteor Ionisation, Phys. Soc. Rep. Prog. Phys.*, **11**, 444.

HEY, J. S., 1949, *Radio Astronomy, Mon. Not. Roy. Astr. Soc.*, **109**, 179.

HOFFMEISTER, C., 1950, *Fundamental problems of Meteoric Astronomy*, *Observatory*, **70**, 70.

LOVELL, A. C. B., 1950, *Meteor Ionisation in the Upper Atmosphere*, *Sci. Prog.*, **38**, 22.

LOVELL, A. C. B., 1951, *The New Science of Radio Astronomy*, *Nature*, **167**, 94.

PORTER, J. G., 1949, *The Study of Meteor Velocities*, *Brit. Astr. Assocn. J.*, **60**, 4.

WHIPPLE, F. L., and others, 1948, *Meteor Astronomy*, *Observatory*, **68**, 226.

C. REFERENCES

ALMOND, M., 1951, *Mon. Not. Roy. Astr. Soc.*, **111**, 37.

ALMOND, M., DAVIES, J. G., and LOVELL, A. C. B., 1950, *Observatory*, **70**, 112; 1951, *Mon. Not. Roy. Astr. Soc.*, **111**. In press.

ASPINALL, A., CLEGG, J. A., and LOVELL, A. C. B., 1949, *Mon. Not. Roy. Astr. Soc.*, **109**, 352.

CLEGG, J. A., and DAVIDSON, I. A., 1950, *Phil. Mag.*, Ser. 7, **41**, 77.

DAVIES, J. G., and ELLYETT, C. D., 1949, *Phil. Mag.*, Ser. 7, **40**, 614.

DAVIES, J. G., and GREENHOW, J. S., 1951, *Mon. Not. Roy. Astr. Soc.*, **111**, 26.

ELLYETT, C. D., 1949, *Mon. Not. Roy. Astr. Soc.*, **109**, 359.

ELLYETT, C. D., 1950, *Phil. Mag.*, Ser. 7, **41**, 694.

GREENHOW, J. S., 1950, *Phil. Mag.*, Ser. 7, **41**, 682.

HEY, J. S., PARSONS, S. J., and STEWART, G. S., 1947, *Mon. Not. Roy. Astr. Soc.*, **107**, 176.

HOFFMEISTER, C., 1951, *Observatory*, **71**, 34.

McKINLEY, D.W.R., 1951, *Astrophys. J.*, **113**, 225.

MILLMAN, P. M., and HOFFLEIT, D., 1937, *Harvard Annals*, **105**, 601.

MILLMAN, P. M., and McKINLEY, D. W. R., 1950, *Observatory*, **70**, 156.

PORTER, J. G., 1944, *Mon. Not. Roy. Astr. Soc.*, **104**, 257.

PRENTICE, J. P. M., 1932, *Brit. Astr. Assocn. J.*, **42**, 243.

SHAPLEY, H., ÖPIK, E. and BOOTHROYD, S. L., 1932, *Proc. Nat. Acad. Sci.*, **18**, No. 1.

WHIPPLE, F. L., 1940, *Proc. Am. Phil. Soc.*, **83**, 711.

WHIPPLE, F. L., 1947, *Proc. Am. Phil. Soc.*, **91**, 189.

WRIGHT, F. W., and WHIPPLE, F. L., 1950, *Harvard. Coll. Tech. Rep.*, No. 6=*Harvard Reprint*, Ser. 2–35.

Appendices

APPENDIX 1

The calculation of an ephemeris

The P's, Q's and R's which are used in computing the co-ordinates of a comet (see equations (54)) may be referred, not only to the equator, but also to any convenient frame of reference. Thus when the ecliptic is being used, the direction cosines of the axes P, Q and R become :—

$$\begin{aligned}
P_x' &= \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i\\
P_y' &= \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i\\
P_z' &= \sin\omega\sin i\\[4pt]
Q_x' &= -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i\\
Q_y' &= -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i\\
Q_z' &= \cos\omega\sin i\\[4pt]
R_x' &= \sin i\sin\Omega\\
R_y' &= -\sin i\cos\Omega\\
R_z' &= \cos i
\end{aligned}$$

In the more usual case, these direction cosines are referred to the plane of the earth's equator, and then become :—

$$\begin{aligned}
P_x &= P_x'\\
P_y &= P_y'\cos\varepsilon - P_z'\sin\varepsilon\\
P_z &= P_y'\sin\varepsilon + P_z'\cos\varepsilon\\[4pt]
Q_x &= Q_x'\\
Q_y &= Q_y'\cos\varepsilon - Q_z'\sin\varepsilon\\
Q_z &= Q_y'\sin\varepsilon + Q_z'\cos\varepsilon\\[4pt]
R_x &= R_x'\\
R_y &= R_y'\cos\varepsilon - R_z'\sin\varepsilon\\
R_z &= R_y'\sin\varepsilon + R_z'\cos\varepsilon
\end{aligned}$$

The calculation of the numerical values of these complicated expressions may be carried out by logarithms or a calculating machine.

A. Using Logarithms

The elements of the orbit must first be corrected (if necessary) to the equinox of the beginning of the year for which the ephemeris is required. The formulae for correcting, ω, Ω and i are given in the *Nautical Almanac*, and may also be found, together with the necessary constants, in the two volumes of *Planetary Co-ordinates*, Table II.

The equatorial constants are computed by logarithms by finding a number of auxiliary quantities U, V, W ; m, M ; A, B, C.

Put

$$U\sin A = \cos\Omega \qquad m\sin M = \sin i.$$
$$U\cos A = -\cos i.\sin\Omega \qquad m\cos M = \cos i.\cos\Omega$$

$$V \sin B = \sin \Omega . \cos \varepsilon \qquad W \sin C = \sin \Omega . \sin \varepsilon$$
$$V \cos B = m \cos (M + \varepsilon) \qquad W \cos C = m \sin (M + \varepsilon)$$

where ε is the obliquity of the ecliptic for the same equinox.

Then if

$$A' = A + \omega \qquad B' = B + \omega \qquad C' = C + \omega$$

the equatorial constants are given by :

$$aP_x = aU. \sin A' \qquad bQ_x = bU. \cos A'$$
$$aP_y = aU. \sin B' \qquad bQ_y = bU. \cos B'$$
$$aP_z = aU. \sin C' \qquad bQ_z = bU. \cos C'$$

where $b = a. \cos \phi$ is computed from the values of a and e ($=\sin \phi$).

Checks—Writing $A_x = aP_x$, $B_x = bQ_x$, and similar expressions for the other constants, a complete check on signs and magnitudes is obtained from

$$A_x . B_x + A_y . B_y + A_z . B_z = 0$$

or, more simply, $\Sigma AB = 0$.

In cases where this check fails, the error may often be found by noting that $\Sigma A^2 = a^2$ and $\Sigma B^2 = b^2$.

The commonest source of error is to be found in the original sines and cosines of the elements. Every possible care should be taken to check these, both as regards sign and magnitude.

The solution of Kepler's equation.—The heliocentric co-ordinates are computed, in logarithmic working, from the equations

$$x = A_x . \cos E + B_x . \sin E - A_x . e$$
$$y = A_y . \cos E + B_y . \sin E - A_y . e$$
$$z = A_z . \cos E + B_z . \sin E - A_z . e$$

in which the eccentric anomaly, E, is found by a solution of Kepler's equation $M = E - e \sin E$ for each date. This is a transcendental equation and must be solved by successive approximations. There are great advantages in this work in using tables in degrees and decimals. With the publication of the new *Chambers's Six-Figure Mathematical Tables**, the computer has no excuse for using inadequate or faulty tables.

It may be shown that if E' is an approximation to E, and if $M' = E' - e. \sin E'$, then a better approximation to E is given by

$$E' + \frac{M - M'}{1 - e . \cos E'}$$

and this new value may again be improved by the same method if necessary. It should be noted that the angles in Kepler's equation are in circular measure, and in order to keep M and E in degrees it is necessary to multiply e by 57·29578 (log = 1·7581226). Calling this product $e°$, Kepler's equation is actually solved in the form

$$M = E - e° . \sin E.$$

In applying the correction, however, it is the normal value of e that is used in $1 - e. \cos E$, and since this quantity is not required to any great accuracy, it may be worked on a slide-rule, or a preliminary table may be constructed to give the value of this quantity (or its logarithm) to 3 figures for different values of E.

The initial approximation to E may be obtained from tables, by slide-rule, or by graphical methods, but since the ephemeris usually covers the period on either side of perihelion passage, it is easier to start at perihelion and work out-

* *Chambers's Six-Figure Mathematical Tables*, edited by Dr. L. J. Comrie, London, 1949. In two volumes : Volume I, Logarithmic values ; Volume II, Natural values.

wards. At perihelion, both M and E are small angles, so that Kepler's equation may be written $M = E - e.E$ and $E = M/(1 - e)$ may be computed for one or two dates on either side of T. Once a few values of E have been formed in this way, they can be tabulated and differenced—usually to the third or fourth difference. The run of the differences enables the computer to make a good estimate of the next value of E. This is corrected as before, and the table continued.

The ephemeris.—The values of sin E and of cos E are now substituted in the equations for the heliocentric co-ordinates, and the solar co-ordinates X, Y, Z are added. The latter are obtained from the *Nautical Almanac*, and once again care must be taken to use the co-ordinates for the correct equinox. The R.A. and dec. of the comet are then computed from the equations.

$$\begin{aligned} \xi &= x + X = \Delta \,.\, \cos\delta \,.\, \cos\alpha \\ \eta &= y + Y = \Delta \,.\, \cos\delta \,.\, \sin\alpha \\ \zeta &= z + Z = \Delta \,.\, \sin\delta \end{aligned}$$

The values of Δ and of r will also be required. The methods which are actually adopted for computing these quantities will be clear from the example and detailed notes which follow.

General.—The calculation of a prolonged piece of work of this kind should always be done in tabular form. If the working of each date is arranged in columns the work is carried out *ALONG THE LINES*, never down the columns. In this way, a great deal of time is saved in using the tables, and there is also a welcome relief from boredom.

As will be seen, the usual method of using positive characteristics for all logarithms is adopted, and the logarithm of a negative quantity is indicated by the letter **n** following. A great deal of time can be saved by using addition and subtraction logarithms, but as these are not at all common (and also have their own peculiar difficulties) they are not used in the example. Various convenient devices for quick working will occur to the computer—for example, the value of a constant quantity can be written on the edge of a slip of paper, which is placed above the line of working to effect addition. This is done, for example, in lines 34, 35 and 36 in order to add log a, log b and log e respectively. The equatorial constants are also added to log sin E and log cos E in this way. Detailed notes follow :

Line 11.—Note the change of sign.

Line 13.—Here the larger of the two quantities sin A or cos A is used, but no thought is required. Find the tangent in the tables, and run the eye along the same line, right or left, select the larger of the two quantities, and write it down.

The quadrant of the angle A (and also of B, C and M) is found from the signs of sine and cosine. Thus the angle A has a negative sine (*line* 6) and a positive cosine (*line* 11) and is therefore in the fourth quadrant.

Line 14.—Subtract *line* 13 from the larger of *lines* 6 or 11. In this way all possible accuracy is retained in the working. Note that the two numbers (*lines* 6 and 13, or 11 and 13) must have the same sign, so that U becomes positive.

The same remarks apply to *lines* 17, 24 and 29. In all such cases, the value of the sine or cosine must be interpolated to correspond with the value of the tangent. The angle is read last.

The quantities U, V, W and m are always positive.

Line 40.—A cometary ephemeris is computed at ten-day intervals, the dates being such that the integral part of the Julian date at midnight is divisible by 10. These dates are readily obtained from the volumes of *Planetary Co-ordinates.*

Line 43.—The values of M are computed from $M = n(t - T)$. They will be negative before perihelion, and positive after it, and so, of course, will E. The accuracy of M should not be forced. Three decimals of a degree are sufficient when working with 5-figure tables, while a fourth place may be retained when using 6-figure tables. This is unnecessarily hard work if a search ephemeris is required.

Line 51.—Slide-rule or preliminary table.

Line 57.—In the cases shown, complete agreement with *line* 44 has been obtained after only two corrections. If the preliminary values of E' are too rough, it may be necessary to repeat *lines* 53–57.

Line 59.—Values of sin E should be marked negative for dates before T. Lines 58 and 59 refer to the value of E in line 53.

Line 62.—The natural value of a is used here. This value of r should be checked for, say, half a dozen dates, by showing that $r^2 = x^2 + y^2 + z^2$, the necessary squares being obtained from *Barlow* or similar tables.

Line 72.—The values of x, y, z are obtained separately as shown, since they are needed for other purposes, e.g., in the check on r, and in calculating the variation. The values of X (and of Y and Z) should not be taken from *Planetary Co-ordinates,* in which the values refer to the centre of gravity of the Earth-Moon system. The solar co-ordinates must refer to the Earth's centre, and should be taken from the *Nautical Almanac.*

Line 90.—As before, the larger of sin or cos is used, corresponding to the value of the tangent in *line* 89 ; at the same time *line* 96 is filled in. *Line* 90 is then subtracted from the larger of 87, 88.

Line 92.—This quantity is log Δ cos δ.

Line 94.—This is another case where the larger of sin or cos is used. It will be obvious that cos is used if the declination is less than 45°—as it is here. If the declination is very large, it is better to use the sine in this line, and subtract from 91, in order to get 95.

Line 96.—The quadrant of α is decided from the signs of 74 (same sign as cos α) and 80 (same sign as sin α).

Line 97.—The sign of δ is always the same as the sign of ζ (line 86).

The conversion of R.A. to hours and minutes is easily accomplished mentally, although conversion tables are given in *Chambers's Six-Figure Tables,* and other works.

The whole ephemeris is finally tabulated, and checked by differencing. It should be found that the differences run quite smoothly.

The example which follows illustrates the various stages in the calculation of an ephemeris of Comet du Toit-Neujmin-Delporte, 1941 VII for 1952 from the elements of R. Luss :—

Epoch 1952 Sept. 25·0 U.T.			T	1952 Sept. 10·3756 U.T.
ω	70°·7901	1950·0	e	0·574945
Ω	228·3713	1950·0	a	3·146089
i	3·2238	1950·0	n	0°·1766231
q	1·33726		P	5·5805 years

In this case the ephemeris, for ten-day dates in 1952, was computed for equinox 1950·0.

ω	70·7901°	1950·0	a	3·146089
☊	228·3713		e	0·574945
i	3·2238			
ε	23·4458			

LOGS

1	a	0·497772		
2	$e = \sin\phi$	9·759626		
3	$\cos\phi$	9·912856		
4	$1 + 3 = b$	0·410628		
5	sin ☊	9·873591n		
6	cos ☊ $= U \sin A$	9·822365n		
7	$\sin i = m \sin M$	8·750016		
8	$\cos i$	9·999312		
9	$\sin \varepsilon$	9·599753		
10	$\cos \varepsilon$	9·962576		
11	$-(5 + 8) = U \cos A$	9·872903		
12	$6 - 11 = \tan A$	9·949462n	A	318·3262°
13	sin or cos A	9·873287	ω	70·7901
14	6 (or 11) $- 13 = U$	9·999616	A'	29·1163
15	$6 + 8 = m \cos M$	9·821677n		
16	$7 - 15 = \tan M$	8·928339n	M	175·1536
17	sin or cos M	9·998444n	ε	23·4458
18	7 (or 15) $- 17 = m$	9·823233	$M + \varepsilon$	198·5994
19	$\sin(M + \varepsilon)$	9·503722n		
20	$\cos(M + \varepsilon)$	9·976704n		
21	$5 + 10 = V \sin B$	9·836167n		
22	$18 + 20 = V \cos B$	9·799937n	B	227·3871
23	$21 - 22 = \tan B$	0·036230	ω	70·7901
24	sin or cos B	9·866845n	B'	298·1772
25	21 (or 22) $- 24 = V$	9·969322		
26	$5 + 9 = W \sin C$	9·473344n		
27	$18 + 19 = W \cos C$	9·326955n	C	234·4786
28	$26 - 27 = \tan C$	0·146389	ω	70·7901
29	sin or cos C	9·910571n	C'	305·2687
30	26 (or 27) $- 29 = W$	9·562773		
31	sin A' B' C'	9·687158	9·945218n	9·911931n
32	U V W	9·999616	9·969322	9·562773
33	cos A' B' C'	9·941330	9·674126	9·761486
34	$1 + 31 + 32 = aP$	0·184546	0·412312n	9·972476n
35	$4 + 32 + 33 = bQ$	0·351574	0·054076	9·734887
36	$2 + 34$	9·944172	0·171938n	9·732102n
37	antilog 36 $= aP.e$	+0·87937	−1·48572	−0·53964
Check:				
38	$34 + 35$	0·536120	0·466388n	9·707363n
39	antilog 38	+3·43653	−2·92677	−0·50976

The sum of these three numbers is zero—an unusually good check. An error of three of four units in the last figure would have been considered satisfactory. It is usual to keep the first two terms of the equations in logarithmic form, while the constant term is used as a natural number. Hence the equations for this comet may be written as follows, the square brackets indicating logarithms :

$$\begin{aligned} x &= [0{\cdot}184546\ ] \cos E + [0{\cdot}351574] \sin E && - 0{\cdot}87937 \\ y &= [0{\cdot}412312\mathbf{n}] \qquad\qquad [0{\cdot}054076] && + 1{\cdot}48572 \\ z &= [9{\cdot}972476\mathbf{n}] \qquad\qquad [9{\cdot}734887] && + 0{\cdot}53964 \end{aligned}$$

In lines 37, 39 and elsewhere, the antilogarithms are found by using the logarithm tables inversely. Indeed, the wise computer never uses tables of antilogarithms ; these are not only a source of mistakes, but are less accurate than logarithm tables used inversely, especially when the logarithm is less than 0·3.

T Sept. 10·3756	$\log e$ = 9·759626
$\log n$ = 9·24705	$\log e^{\circ}$ = 1·51775
a = 3·14609	$\log ae$ = 0·25740

40	t (1952)	Sept. 5	Sept. 15	Sept 25 . . .	Dec. 4
41	$t - T$	−5·3756	+4·6244	+14·6244	+84·6244
42	log 41	0·73043**n**	0·66506	1·16508	1·92750
43	$\log M$	9·97748**n**	9·91211	0·41213	1·17455
44	M	−0·949	+0·817	+ 2·583	+14·947
45	E'	−2·23	+1·92	+ 6·08	+32·7
46	$\log \sin E'$	8·59007**n**	8·52510	9·02497	9·73259
47	46 + $\log e^{\circ}$	0·10782**n**	0·04285	0·54272	1·25034
48	antilog 47	−1·282	+1·104	+ 3·489	+17·797
49	45–48	−0·948	+0·816	+ 2·591	+14·903
50	44–49	−0·001	+0·001	− 0·008	+ 0·044
51	$1 - e \cos E$	0·425	0·426	0·432	0·516
52	50 ÷ 51	−0·002	+0·002	− 0·019	+ 0·085
53	45 + 52 = E''	−2·232	+1·922	+ 6·061	+32·785
54	$\log \sin E''$	8·59046**n**	8·52555	9·02361	9·73359
55	54 + $\log e^{\circ}$	0·10821**n**	0·04330	0·54136	1·25134
56	antilog 55	−1·283	+1·105	+ 3·478	+17·838
57	53–56	−0·949	+0·817	+ 2·583	+14·947
58	$\cos E$	9·99967	9·99976	9·99756	9·92465
59	$\sin E$	8·59046**n**	8·52555	9·02361	9·73359
60	58 + $\log ae$	0·25707	0·25716	0·25496	0·18205
61	antilog 60	1·8075	1·8078	1·7987	1·5207
62	a − 62 = r	1·3386	1·3383	1·3474	1·6254
63	58 + aP_x	0·18422	0·18431	0·18211	0·10920
64	59 + bQ_x	8·94203**n**	8·87712	9·37518	0·08516
65	58 + aP_y	0·41198**n**	0·41207**n**	0·40987**n**	0·33696**n**
66	59 + bQ_y	8·64454**n**	8·57963	9·07769	9·78767
67	58 + aP_z	9·97215**n**	9·97224**n**	9·97004**n**	9·89713**n**
68	59 + bQ_z	8·32535**n**	8·26044**n**	8·75850	9·46848

69	antilog 63	+1·5283	+1·5287	+1·5209	+1·2859
70	antilog 64	−0·0875	+0·0754	+0·2372	+1·2166
71	constant	−0·8794	−0·8794	−0·8794	−0·8794
72	sum = x	+0·5614	+0·7247	+0·8787	+1·6231
73	X	−0·9605	−0·9958	−1·0022	−0·3086
74	sum = ξ	−0·3991	−0·2711	−0·1235	+1·3145
75	antilog 65	−2·5821	−2·5827	−2·5696	−2·1725
76	antilog 66	−0·0441	+0·0380	+0·1196	+0·6133
77	constant	+1·4857	+1·4857	+1·4857	+1·4857
78	sum = y	−1·1405	−1·0590	−0·9643	−0·0735
79	Y	+0·2807	+0·1276	−0·0293	−0·8586
80	sum = η	−0·8598	−0·9314	−0·9936	−0·9321
81	antilog 67	−0·9379	−0·9381	−0·9333	−0·7891
82	antilog 68	−0·0212	+0·0182	+0·0574	+0·2941
83	constant	+0·5396	+0·5396	+0·5396	+0·5396
84	sum = z	−0·4195	−0·3803	0·3363	+0·0446
85	Z	+0·1218	+0·0553	0·0127	−0·3724
86	sum = ζ	−0·2977	−0·3250	0·3490	−0·3278
87	log 80	9·93440**n**	9·96914**n**	9·99721**n**	9·96946**n**
88	log 74	9·60108**n**	9·43313**n**	9·09167**n**	0·11876
89	87–88 = tan α	0·33332	0·53601	0·90554	9·85070**n**
90	sin (cos) α	9·95763**n**	9·98234**n**	9·99667**n**	9·91155
91	log 86	9·47378**n**	9·51188**n**	9·54282**n**	9·51561**n**
92	87 (or 88)–90	9·97677	9·98680	0·00054	0·20721
93	91–92 = tan δ	9·49701**n**	9·52508**n**	9·54228**n**	9·30840**n**
94	sin (cos) δ	9·97957	9·97693	9·97510	9·99119
95	91 (or 92)–94	9·99720	0·00987	0·02544	0·21602
96	α	$16^h\ 20^m{\cdot}4$	$16^h\ 55^m{\cdot}1$	$17^h\ 31^m{\cdot}7$	$21^h\ 38^m{\cdot}6$
97	δ	−17° 26′	−18° 31′	−19° 13′	−11° 30′
98	antilog 95 = Δ	0·9936	1·0230	1·0603	1·6444

B. Using a Calculating Machine

Much of the advice given above applies equally to machine working ; and the following notes deal only with points of difference.

The equatorial constants are best computed by the device given by Merton : Write down the sines and cosines of the angles ω, Ω, i, and of the obliquity ε (the latter from the relevant *Nautical Almanac* or the volumes of *Planetary Co-ordinates*). These quantities should be thoroughly checked, since it is easy to make a mistake over signs, or to read sine for cosine. Check also that $\sin^2 A + \cos^2 A = 1$. The values of the constants are then found by computing, in the order given :

$$\begin{array}{llll}
(1) = \sin\omega \, . \sin \Omega & & (4) = \cos\omega . \sin\Omega & \\
(2) = \sin\omega \, . \cos \Omega & & (5) = \cos\omega . \cos\Omega & \\
(3) = \sin\omega \, . \sin i & & (6) = \cos\omega \, . \sin i & \\
(7) = (5) - (1)\cos i & = P_x & (9) = -(2) - (4)\cos i & = Q_x \\
(8) = (5)\cos i - (1) & & (10) = (2)\cos i + (4) & \\
(11) = (10)\cos\varepsilon - (3)\sin\varepsilon & = P_y & (13) = (8)\cos\varepsilon - (6)\sin\varepsilon & = Q_y \\
(12) = (3)\cos\varepsilon + (10)\sin\varepsilon & = P_z & (14) = (6)\cos\varepsilon + (8)\sin\varepsilon & = Q_z
\end{array}$$

These quantities are checked, as usual, by showing that

$$\Sigma PQ = 0$$

and agreement should be found within one or two units of the last decimal used. In order to avoid rounding-off errors, an extra decimal may be retained in all the intermediate quantities, so that the final values of $A_x = a.P_x$ etc., may be as accurate as possible.

For machine work it is convenient to use the equatorial equations in the form

$$x = A_x(\cos E - e) + B_x \sin E$$

and the values of E are so easy to find in the machine process that no difficulty is likely to arise. Initial values in the neighbourhood of T may be found if desired, as explained above, and in any case, the table of values of E will be found useful.

Compute first $e^\circ = 57{\cdot}29578e$, and with this product on the setting levers (S.L.) arrange the value of M in the product register (P.R.). (The Brunsviga 20 is a particularly convenient machine for this purpose, since thumb-wheels are provided for such individual settings.) With any suitable value of E, multiply by $\sin E$. The P.R. then shows a better approximation to E, and the process is repeated. The method uses Kepler's equation in the form

$$E = M + e^\circ . \sin E$$

and the work continues until the value of E in the P.R. corresponds exactly with the value of $\sin E$ in the multiplier register (M.R.).

The value of $\sin E$ is noted (line 7 in the example that follows) and $\cos E$ is interpolated to correspond with $\sin E$, not from a reading of E. The value of E, in fact, is not wanted, except to give an estimate of the value on the next date.

The essence of machine work is to write as little as possible ; hence all the x's are computed together, then the y's and the z's. In other words, the computer should work along the lines, as usual. The r's are found by setting a on the P.R. and ae on the S.L., and multiplying by successive values of $\cos E$ in a backward direction. The R.A. is found as in the logarithmic method, but using either

$$\tan\alpha = \eta/\xi \qquad \text{or } \cot\alpha = \xi/\eta$$

whichever is numerically less than 1. It is convenient to determine the R.A. from the 7-figure tables with argument in time.* The values of Δ and of δ are best determined by computing

$$\Delta^2 = \xi^2 + \eta^2 + \zeta^2$$

the square root being found by the normal machine method of dividing by an approximate root (*Barlow's Tables*,† using 4 figures only). The true root (to

* *Seven-Figure Trigonometrical Tables for every second of time*, H.M. Stationery Office, London, 1939.

† *Barlow's Tables of Squares, Cubes, Square-roots, etc.*, Spon, London, various editions.

twice the number of figures used in the approximate value) is simply the mean of the approximate root and the quotient, and is easily read mentally from the M.R. Then

$$\sin \delta = \zeta / \Delta$$

the sign of δ being the same as that of ζ.

The calculation of the variation is not illustrated. It is carried out in exactly the same way, but this time combining the values of X, Y, Z for the given date with the x, y, z, for the previous ten-day date. The R.A. and dec. calculated in this way will give the values that the comet would have if T were 10 days later. Subtract the true ephemeris values and divide by 10 to obtain the variation.

The example which follows (using the same comet) has been computed to illustrate rather more accurate work than is normally necessary in a search ephemeris. The equatorial constants have been obtained with the help of *Peters*' 7-figure tables,* and the rest of the work has made use of the new *Chambers's Six-Figure Tables*, Volume II.

$$\left.\begin{array}{ll} \omega & 70^\circ\!\cdot\!7901 \\ \Omega & 228\cdot3713 \\ i & 3\cdot2238 \end{array}\right\} \; 1950\cdot0 \qquad \begin{array}{rcl} e = \sin\phi & = & 0\cdot574945 \\ \cos\phi & = & 0\cdot8181921 \\ a & = & 3\cdot146089 \\ b & = & 2\cdot574105 \end{array}$$

	ω	Ω	i	ε
sin	+9443196	−7474654	+0562362	+39788118
cos	+3290298	−6643007	+9984175	+91743695

(1)	−70584623		(4)	−24593839	
(2)	−62731217		(5)	−21857473	
(3)	+05310494		(6)	+01850339	
(7)	+48615450	$= P_x$	(9)	+87286136	$= Q_x$
(8)	+48761739		(10)	−87225784	
(11)	−82137103	$= P_y$	(13)	+43999686	$= Q_y$
(12)	−29833454	$= P_z$	(14)	+21098948	$= Q_z$

Check : $\Sigma PQ = +1 \times 10^{-8}$

$$\begin{array}{lllll} Ax = a\,.\,P_x = +1\cdot529485 & \qquad & B_x = b\,.\,Q_x = +2\cdot246837 \\ Ay = a\,.\,P_y = -2\cdot584106 & & B_y = b\,.\,Q_y = +1\cdot132596 \\ Az = a\,.\,P_z = -0\cdot938587 & & B_z = b\,.\,Q_z = +0\cdot543109 \end{array}$$

Check : $\Sigma AB = +3 \times 10^{-7}$

* Peters, *Siebenstellige Werte der trigonometrischen Funktionen von Tausendstel zu Tausendstel des Grades*, Berlin, 1918.

T	Sept. 10·3756	e	0·574945
n	0°·1766231	e°	32·941922
a	3·146089	ae	1·808828

1	t (1952)	Sept. 5	Sept. 15	Sept. 25 . . .	Dec. 4
2	$t - T$	−5·3756	+4·6244	+14·6244	+84·6244
3	M	−0·9495	+0·8168	+2·5830	+14·9466
4	E	−2·2330	+1·9211	+6·0616	+32·7835
5	cos E	+0·999241	+0·999438	+0·994409	+0·840723
6	cos $E - e$	+0·424296	+0·424493	+0·419454	+0·265778
7	sin E	−0·038963	+0·033524	+0·105598	+0·541466
8	x	+0·561410	+0·724579	+0·878810	+1·623089
9	X	−0·960508	−0·995850	−1·002217	−0·308604
10	sum $= \xi$	−0·399098	−0·271271	−0·123407	+1·314485
11	y	−1·140555	−1·058966	−0·964314	−0·073537
12	Y	+0·280748	+0·127605	−0·029314	−0·858620
13	sum $= \eta$	−0·859807	−0·931361	−0·993628	−0·932157
14	z	−0·419400	−0·380217	−0·336343	+0·044619
15	Z	+0·121758	+0·055343	−0·012714	−0·372363
16	sum $= \zeta$	−0·297642	−0·324874	−0·349057	−0·327744
17	α	$16^h\ 20^m{\cdot}40$	$16^h\ 55^m{\cdot}05$	$17^h\ 31^m{\cdot}68$	$21^h\ 38^m{\cdot}63$
18	$\triangle$	0·993548	1·023017	1·060362	1·644446
19	δ	−17° 25′·9	−18° 31′·0	−19° 13′·1	−11° 29′·7
20	r	1·338634	1·338278	1·347374	1·625366
21	r^2	1·791941			2·641815
22	$x^2 + y^2 + z^2$	1·791943			2·641816

APPENDIX 2

Conversion of R.A. and Dec.

In meteor calculations it is necessary to convert the R.A. and dec. of the radiant into longitude and latitude, or to obtain the zenith distance of the radiant. Conversions from one system to another may readily be performed with a small calculating machine as follows :—

(*a*) *R.A. and dec. to longitude-latitude*

Put
$$l = \cos\delta\cos\alpha$$
$$m = \cos\delta\sin\alpha$$
$$n = \sin\delta$$

Then
$$\cos\beta\cos\lambda = l$$
$$\cos\beta\sin\lambda = n\sin\varepsilon + m\cos\varepsilon$$
$$\sin\beta = n\cos\varepsilon - m\sin\varepsilon$$

(*b*) *Long.-lat. to R.A. and dec.*

Put
$$L = \cos\beta\cos\lambda$$
$$M = \cos\beta\sin\lambda$$
$$N = \sin\beta$$

Then
$$\cos\delta\cos\alpha = L$$
$$\cos\delta\sin\alpha = M\cos\varepsilon - N\sin\varepsilon$$
$$\sin\delta = M\sin\varepsilon + N\cos\varepsilon$$

(*c*) *R.A. and dec. to zenith distance*

If t is the sidereal time, the hour-angle h is given by $t - \alpha$. The zenith distance may be obtained from tables, (e.g., navigation tables) or may be computed from

$$\cos z = \sin\delta\sin\phi - \cos\delta\cos\phi\cos h$$

where ϕ is the latitude of the observer. The correction for zenith attraction may then be made as follows :

Compute the parallactic angle π from

$$\sin\pi = \frac{\cos\phi.\sin h}{\sin z}$$

and take the value of π as being less than 90°. Then the corrections in R.A. and dec. are given by

$$\Delta\alpha = \Delta z.\sin\pi\sec\delta$$
$$\Delta\delta = \Delta z.\cos\pi$$

The correction $\Delta\alpha$ always has the same sign as $\sin h$.

The correction $\Delta\delta$ is always negative, except in those rare cases where the radiant lies between the pole and the zenith, i.e., when

$\sin\delta\cos z$ is greater than $\sin\phi$.

In such cases $\Delta\delta$ is positive, and care must be taken to apply this correction with the correct sign to all observations in which z is less than the co-latitude.

TABLE 21

Examples.

	Date	May 4·8	July 28·9	Oct. 9·9	Dec. 14·3	Dec. 16·1
1	L (1950·0)	43·8	125·2	196·0	261·7	263·5
2	D	−0·8	+0·4	+1·0	+0·3	+0·3
3	sin D	−0142	+0065	+0166	+0058	+0053
4	R	1·0086	1·0153	0·9986	0·9843	0·9841
5	α (1950·0)	336°·0	87°	262°	129°·3	277°·2
6	δ	−0·8	+38	+54	+16·3	−35·0
7	λ	337·4	87·6	248·9	127·4	275·9
8	β	+8·5	+14·6	+76·9	−2·1	−11·7
9	A	313·0	35·6	107·0	172·0	173·8
10	$\lambda - A$	24·4	52·0	141·9	315·4	102·1
11	ε	25·75	53·42	100·31	44·64	101·84
12	sin γ	3401	3134	9898	0521	2072
13	cos γ	9404	9497	1425	−9986	9783
14	V_E	0·9914	0·9848	1·0014		1·0159
15	V	1·3885	0·923	1·3104		1·3400
16	V_E/V	0·7140	1·0508	0·7642	0·7126	0·7581
17	$\varepsilon' - \varepsilon$	18·07	57·54	48·76	30·05	47·90
18	ε'	43·82	110·96	149·07	74·69	149·74
19	F	2355	2927	5088	0503	1044
20	G	6511	8868	0732	−9631	4930
21	H	7215	−3577	−8578	2640	−8638
22	B	−7307	3635	8590	−2696	8664
23	C	6409	8845	0590	−9616	4884
24	S	7676	4665	9983	2745	8726
25	$\sqrt{p}$	1·0750	0·4372	1·3063	0·3851	1·1507
26	p	1·1556	0·1911	1·7064	0·1483	1·3241
27	e sin v	9566	3569	1010	−5278	7531
28	e cos v	1457	−8118	7088	−8493	3455
29	sin i	3068	6274	5097	1832	1196
30	cos i	−9519	7792	8605	−9822	9929
31	v	81°·34	156°·27	8°·11	211°·86	65°·36
32	ω	98·7	23·7	171·9	148·1	294·6
33	Ω	43·8	125·2	196·0	81·7	83·5
34	i	162·1	38·9	30·6	169·4	6·9
35	e	0·968	0·887	0·716	1·0	0·829
36	q	0·575	0·101	0·995	0·074	0·72
	Notes	Aquarids-assuming a (for Halley's comet) = 18	Bouska and Plavec, I.A.U.C. 1251, with V given as 0·923	Giacobinids, assuming a = 3·5	Parabolic. (Comet 1931 IV)	Comet 1881 V assumed a = 4·2

APPENDIX 3

Calculation of meteor orbits.

The method of computing an orbit of a meteor stream is shown in examples opposite, while the following notes explain the various steps of the work.

Lines

1–4. For the given date and time, find L and R from the *Nautical Almanac*, and use Table 22 to interpolate D and $\sin D$.

5–8. The R.A. and dec. of the corrected radiant (α, δ) are converted to longitude and latitude (λ, β).

9, 10. $A = L - 90° + D$

11–13.
$$\begin{aligned} \cos \varepsilon &= \cos \beta \cos (\lambda - A) \\ \sin \varepsilon \cos \gamma &= \cos \beta \sin (\lambda - A) \\ \sin \varepsilon \sin \gamma &= \sin \beta \qquad \text{(always positive)} \end{aligned}$$

14. From Table 22.

15, 16. For values of V, see page 86.

17, 18. $\sin (\varepsilon' - \varepsilon) = V_E/V \sin \varepsilon$
For methods in other cases, see page 86.

19–21.
$$\begin{aligned} F &= \cos \varepsilon' \\ G &= \sin \varepsilon' \cos \gamma \\ H &= \sin \varepsilon' \sin \gamma \end{aligned}$$

22–24.
$$\begin{aligned} B &= G \sin D - F \\ C &= G + F \sin D \\ S^2 &= 1 - C^2 \end{aligned}$$
S is most easily found as the sine of the angle whose cosine is C.

25, 26.
27, 28.
$$\begin{aligned} \sqrt{p} &= VRS \\ e \sin v &= CV \sqrt{p} \\ e \cos v &= p/r - 1 \end{aligned}$$
For a parabola these simplify to $\tan v/2 = C/S$;
e may be checked from $e = \sqrt{1 - p/a}$

29, 30.
$$\begin{aligned} \sin i &= H/S \text{ (always positive)} \\ \cos i &= B/S \end{aligned}$$
i is less than 90° if ε' is greater than 90°.

31. From 27, 28

32, 33. If the radiant is *North* of the ecliptic,
$$\begin{aligned} \Omega &= L \\ \omega &= 180° - v \end{aligned}$$
If the radiant is *South* :
$$\begin{aligned} \Omega &= L \pm 180° \\ \omega &= 360° - v \end{aligned}$$

34. From 29–30.

35. From 27–28.

36. $q = a(1 - e) = p/(1 + e)$
In the case of a parabola, $q = \frac{1}{2}p$

The five examples given in Table 21 illustrate the application of the method under different conditions, and with angles in different quadrants.

TABLE 22

L	$\sin D$	D	V_E	$\sqrt{1 - R/2}$
°		°		
0	− 0·0164	+ 0·94	1·0038	0·7084
10	− 0·0167	− 0·96	1·0009	0·7074
20	166	95	0·9980	7064
30	159	91	9951	7054
40	148	85	9924	7044
50	132	76	9899	7035
60	− 0·0112	− 0·64	0·9878	0·7027
70	89	51	9860	7021
80	63	36	9846	7016
90	35	20	9837	7013
100	− 0·0006	− 0·03	9834	7012
110	+ 0·0023	+ 0·13	0·9835	0·7012
120	51	29	9842	7015
130	78	45	9854	7019
140	103	59	9870	7025
150	124	71	9890	7032
160	+ 0·0142	+ 0·81	0·9913	0·7040
170	155	89	9940	7050
180	164	94	9968	7060
190	167	96	0·9997	7070
200	166	95	1·0025	7080
210	+ 0·0159	+ 0·91	1·0054	0·7090
220	148	85	0080	7099
230	132	76	0105	7108
240	112	64	0126	7115
250	89	51	0143	7121
260	+ 0·0063	+ 0·36	1·0156	0·7126
270	35	20	0165	7129
280	+ 0·0006	+ 0·03	0169	7130
290	− 0·0023	− 0·13	0166	7130
300	51	29	0160	7127
310	− 0·0078	− 0·45	1·0149	0·7123
320	103	59	0134	7118
330	124	71	0115	7111
340	142	81	0091	7103
350	155	89	0066	7094
360	− 0·0164	− 0·94	1·0038	0·7084

Index